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CONCRETE AND CONSTRUCTIONAL ENGINEERING

INCLUDING PRESTRESSED CONCRETE

Volume LIV, No. 9. LONDON, SEPTEMBER, 1959.

EDITORIAL NOTES

A Word is Wanted: A Competition.

IN the concrete industry the use of the word reinforcement to describe the steel in reinforced concrete is now so well established that it would be misleading to use it for any other purpose, such as to describe the tensioned steel in prestressed concrete. It may be that in the early experiments the word reinforcement was used to describe materials other than steel, such as wood and iron, that were incorporated in concrete in attempts to provide tensile strength, but nowadays it means steel only unless another material, for example bamboo, is specifically mentioned. Although steel is a ferrous material the term ferro-concrete that was commonly used earlier in the century was a misnomer because it implied that the material comprised iron and concrete. The French term ciment-armé, which was also often used in a translated form armoured concrete, is also misleading because the concrete is not protected as is implied by the word armoured; one would expect armoured concrete to be concrete encased in another tough material. Some of the terms that have confused the terminology of concrete in the past fifty years or so have fallen out of use, but new ones are constantly being invented. An example is the use of the word " shell ", which was imported from Germany, to describe thin curved slabs or domes supported on longitudinal or circular beams, and which is now used to describe thin slabs of any shape that appeals to the architectural exhibitionist. With prestressed concrete came other terms which were quite ridiculous or misleading, such as pretensioned or post-tensioned beams to which only compression (or precompression) has been applied.

We are now confronted with the problem of finding a generic term that can be used to describe the material or device used to produce a state of compression in prestressed concrete. In some early prestressed concrete very thin wires were used, and it became common to describe the steel component as piano wire. Other pioneers used thicker wires, and these are rightly called wires. Later, bars up to more than an inch in diameter were more extensively used, and these are called bars. Still later stranded wire, or wire ropes, have been used, and these are known as strands or stranded wire.

An informative description of concrete reinforced with bars must state whether they are of mild steel or high-tensile steel, whether they are round or square or twisted, whether they have plain or deformed surfaces, and so on; such a description must also state whether the reinforcement is in the form of a

mesh, and if so whether it is an expanded metal or a welded or woven mesh. But the one word reinforcement includes all those types of steel bar and mesh and any other material used for the sake of its tensile strength.

In the case of prestressed concrete we have no such term that includes wires, bars, wire ropes, or other material or device used for the main purpose of putting the concrete into a state of compression. It is becoming common to use the words prestressing steel, but if it is not to be overtaken by developments the word steel should be avoided in an age that has seen metals and other materials replaced by plastic for so many purposes, and which has seen nylon replace hemp for ropes. Thin filaments of glass fibre coated and bound together with polyester resin are already used for the reinforcement of concrete pipes to carry water under pressure, and it is stated that in a U.S.A. laboratory fine filaments of this material have been made with a tensile strength equivalent to more than a million pounds per square inch. Metal scaffolding is a case where the description first used is now misleading; the first metal scaffolding was constructed with steel tubes and so became known as steel scaffolding, with the result that much of the scaffolding now called steel is in fact an alloy of aluminium. Machines known as cement guns are used for applying concrete with aggregate up to $\frac{3}{8}$ in. more often than neat cement slurry. When the pressure of water was used to operate a jack the apparatus was correctly called an hydraulic jack, but this term is now commonly used to describe jacks operated by the pressure of oil, grout, and other liquids. Not only is this wrong, but it has the disadvantage that, like steel scaffolding and cement gun, it gives no indication of the material actually used.

There is now a tendency to introduce into this country the German practice of describing the steel in prestressed concrete as tendons. This anatomical term is obviously wrong, for the tension in a tendon that connects muscle and bone varies with the energy transmitted to it by a muscle, whereas it is essential that the tension in the steel in a prestressed member remain fairly constant after the force of the jack or other stretching device is removed. If the steel in prestressed concrete were to be analogous to tendons it would be necessary to provide permanent sources of energy that would increase and decrease the tension in the steel as the load on the members varied.

In the hope that a word may be found that will be acceptable as a description of the bars, wires, strands, or any other material or device used to prestress concrete, in the same way that the word reinforcement is used to describe any material used to provide tensile resistance in ordinary reinforced concrete, Concrete Publications Limited offer a prize of £25 in a competition for suggestions. The only stipulation is that the words steel and tie be not used, as the word steel would not be comprehensive if other materials are used in the future and tie is already used in another sense in construction. Words already in use may be recommended so long as they are not trade names. The assessor will be an authority on the use of words who is not connected with the concrete industry. Entries may be sent on a postcard or an air-mail form and must reach this office (14 Dartmouth Street, London, S.W.1) by December 31, 1959. The prize will be awarded to the competitor whose suggestion is deemed by the assessor to be the best of those submitted. Any or all of the suggestions may be published in this Journal at the discretion of the Editor.

The Design of Corner Columns.

By R. J. BARTLETT.

THE load-factor method of designing rectangular reinforced concrete members subjected to bending and axial stresses described in this journal for August, 1957, may be extended to the design of such members as corner columns subjected to axial load and bending in two directions, usually at right angles. The following notes relate to the effect on a rectangular section subjected to an axial load P and a bending moment M_x about one axis of symmetry XX when a second bending moment M_y about the axis YY is imposed. The notation, the expressions for the ultimate and yield stresses, and the examples, modified as necessary, are as given in the previous article.

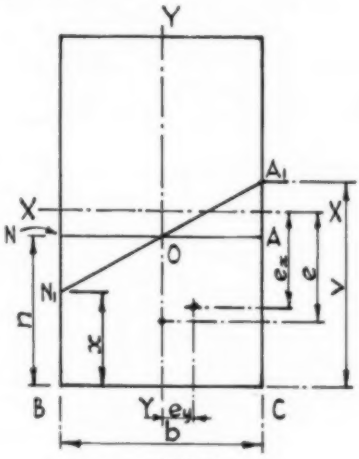


Fig. 1.

Consider an unreinforced section for which the greatest permissible eccentricity is given by $e = \frac{D}{2} - \frac{2P}{3cb}$. If n equals $D - 2e$, then n also equals $\frac{4P}{3cb}$. If the load has two eccentricities e_x and e_y, as in *Fig.* 1, the effective load-bearing part of the section will be trapezoidal, the load P being central over the stress-block. Thus the centroid of the trapezium N_1BCA_1 will be situated at distances e_x and e_y from the axes XX and YY, and its area will equal that of the rectangle NBCA in which $AC = n = D - 2e$.

If $N_1B = x$ and $A_1C = v$, then, since areas N_1BCA_1 and NBCA are equal for a given value of P, $b\left(\frac{x+v}{2}\right) = b.n$, and $x + v = 2n = \frac{8P}{3cb}$. The distance of the centroid of N_1BCA_1 from axis YY is given by

$$e_y = \frac{\frac{b(v-x)}{2} \cdot \frac{b}{6}}{\frac{b(v+x)}{2}} = \frac{b}{6}\left(\frac{v-x}{v+x}\right) = \frac{b}{6}\left(\frac{n-x}{n}\right), \text{ since } 2n = v + x.$$

Therefore $x = n\left(\frac{b - 6e_y}{b}\right) = n\left(1 - \frac{6e_y}{b}\right)$.

The distance of the centroid of N_1BCA_1 from BC is given by

$$\frac{D}{2} - e_x = \frac{\frac{bx^2}{2} + \frac{b(v-x)}{2}\cdot\left(x + \frac{v-x}{3}\right)}{\frac{b(v+x)}{2}} = \frac{x^2 + xv + v^2}{3(v+x)} = \frac{4n^2 - 2nx + x^2}{6n},$$

since $v = 2n\text{-}x$. Since $x = n\left(1 - \frac{6e_y}{b}\right)$, $\frac{D}{2} - e_x = \frac{n}{2}\left(1 + \frac{12e_y^2}{b^2}\right)$.

Therefore
$$e_y^2 = \frac{b^2}{12}\left[\frac{2}{n}\left(\frac{D}{2} - e_x\right) - 1\right] \quad . \quad . \quad . \quad . \quad (1)$$

EXAMPLE I.—A concrete section is 9 in. wide by 12 in. deep; $c = 1000$ lb. per square inch, $P = 50{,}000$ lb., and $M_x = 100{,}000$ in.-lb. It has been shown that $e = 2{\cdot}29$ in. Calculate the greatest permissible moment about axis YY.

$$n = D - 2e = 12 - 4{\cdot}58 = 7{\cdot}42 \text{ in.} \quad e_x = \frac{100{,}000}{50{,}000} = 2 \text{ in.}$$

From formula (1) $e_y^2 = \frac{9^2}{12}\left[\frac{2}{7{\cdot}42}(6 - 2) - 1\right] = 0{\cdot}526.$

Therefore
$$e_y = \sqrt{0{\cdot}526} = 0{\cdot}725 \text{ in.}$$

The greatest permissible moment about YY is $50{,}000 \times 0{\cdot}725 = 36{,}250$ in.-lb.

From formula (1), if e_x and e_y are known,

$$n = \frac{b^2(D - 2e_x)}{12e_y^2 + b^2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Also $n = \frac{4P}{3cb}$, and hence the greatest permissible load

$$P = \frac{3cbn}{4} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which n is as given in formula (2).

If M_x and M_y are known, such that $M_y = k.M_x$, then from formulæ (2) and (3) and the relationship $M_x = P.e_x$

$$e_x = \frac{D \pm \sqrt{D^2 - \frac{4}{3}(NK^2 + 1)Nb^2}}{4(NK^2 + 1)} \quad . \quad . \quad . \quad (4)$$

in which $N = \frac{8M_x}{cb^3}$.

This formula gives two values for e_x, the larger being the greatest permissible eccentricity and the smaller giving the greatest permissible load if substituted in

$$P = \frac{M_x}{e_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

EXAMPLE II.—As Example I, but M_x=100,000 in.-lb. and M_y=25,000 in.-lb. Calculate the greatest permissible load P.

$$k = \frac{25{,}000}{100{,}000} = 0{\cdot}25. \quad N = \frac{8 \times 100{,}000}{1000 \times 9^3} = 1{\cdot}097.$$

$$e_x = \frac{12 \pm \sqrt{12^2 - \frac{4}{3} \times 1{\cdot}0685 \times 1{\cdot}097 \times 9^2}}{4 \times 1{\cdot}0685} = 3{\cdot}68 \text{ in. and } 1{\cdot}94 \text{ in.}$$

Therefore $P = \frac{100{,}000}{1{\cdot}94} = 51{,}600$ lb.

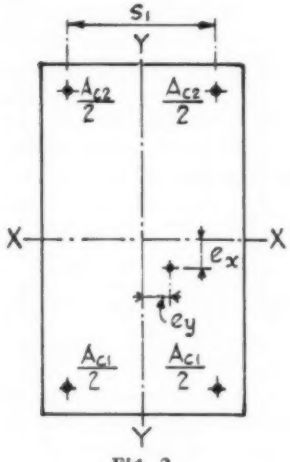


Fig. 2.

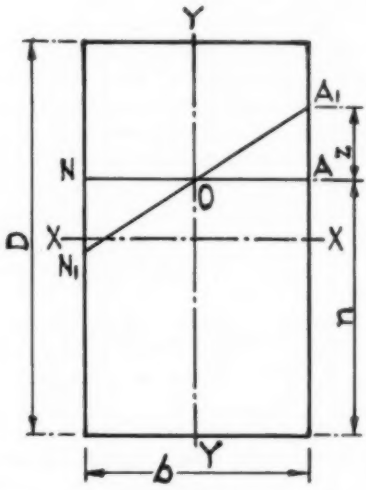


Fig. 3.

In a reinforced section, if P is large and the eccentricities are small the section may remain entirely in compression. The reinforcement A_{c_1} and A_{c_2} should be calculated considering P and M_x only; the areas A_{c_1} and A_{c_2} would usually be as shown in *Fig.* 2. If P also has an eccentricity e_y, the moment $P.e_y$ may be resisted by adding reinforcement A_{c_3} to the reinforcement on the same side of YY as the eccentricity e_y, and subtracting the same amount from the reinforcement on the other side of YY, such that

$$A_{c_3}.t_c.s_1 = P.e_y \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This formula is valid when A_{c_3} does not exceed $\frac{A_{c_1} + A_{c_2}}{2}$. The total reinforcement $(A_{c_1} + A_{c_2})$ is not affected by M_y.

EXAMPLE III.—It has been shown for a section as in Example I, but with P=120,000 lb. and M_x=100,000 in.-lb., that A_{c_1}=1·7 sq. in. and A_{c_2}=0·464 sq. in. Calculate the arrangement of the reinforcement required to resist an additional moment M_y of 36,000 in.-lb.

If the distance s_1 is 6 in., $A_{c_3} \times 18{,}000 \times 6 = 36{,}000$; therefore A_{c_3}=0·333 sq. in. If this is shared equally between A_{c_1} and A_{c_2} then A_{c_1} becomes $\frac{1}{2}(1{\cdot}7 + 0{\cdot}333)$ and $\frac{1}{2}(1{\cdot}7 - 0{\cdot}333)$, that is 1·0165 sq. in. and 0·6835 sq. in. Similarly A_{c_2} becomes

$\frac{1}{2}(0{\cdot}464 + 0{\cdot}333)$ and $\frac{1}{2}(0{\cdot}464 - 0{\cdot}333)$, that is 0·3985 sq. in. and 0·0655 sq. in.

If the section is not entirely in compression, the position of the neutral axis and the quantity of reinforcement should be calculated for P and M_x only. The stress-block must then be adjusted to allow for M_y; that is NA is moved to N_1A_1 as in *Fig.* 3.

Moment M_y is resisted by the couple formed by the addition of OAA_1 to, and the deduction of ONN_1 from, the stress-block; the centroid of each of these areas is $\frac{b}{3}$ from YY.

Therefore, since $OAA_1 = ONN_1$,

$$M_y = \text{stress-block } OAA_1 \times \frac{2b}{3} = \tfrac{3}{4}c \times \frac{z}{2} \times \frac{b}{2} \times \frac{2b}{3}, \text{ and } z = \frac{8M_y}{cb^2}.$$

The reinforcement calculated previously for P and M_x only should now be adjusted to allow for the eccentricity about the axis YY of the centroid of the stress-block N_1A_1CB. The consequent reduction in the moment of resistance of the concrete about the axis XX must be allowed for by increasing the tensile reinforcement and increasing or introducing compressive reinforcement.

The reduction in the moment of resistance of the concrete is equal to the

$$\text{stress-block} \quad OAA_1 \times \frac{AA_1 + NN_1}{3} = \tfrac{3}{4}c.\frac{z}{2}.\frac{b}{2}.\frac{2z}{3} = \frac{cb}{8}.z^2 = \frac{cb}{8}.\frac{8^2M_y^2}{c^2b^4} = \frac{8M_y^2}{cb^3}.$$

The moment of resistance provided by the additional compressive steel A_{ca} is $A_{ca}.t_c.s$

Therefore
$$A_{ca} = \frac{cbz^2}{8t_cs} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in which s is the distance between centres of A_c and A_t.

The additional tensile steel A_{ta} is given by

$$A_{ta} = A_{ca} \times \frac{t_c}{t} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The reinforcement should be placed symmetrically about YY.

EXAMPLE IV.—A section as in Example I, with $P = 15{,}000$ lb., $M_x = 150{,}000$ in.-lb., $n = 3{\cdot}725$ in., $A_t = 0{\cdot}565$ sq. in., and $t = 18{,}000$ lb. per square inch. Calculate the additional reinforcement required if a moment $M_y = 15{,}000$ in.-lb. is imposed; $s = 9$ in. $z = \dfrac{8 \times 15{,}000}{1000 \times 9^2} = 1{\cdot}48$ in.

$$A_{ca} = \frac{1000 \times 9 \times 1{\cdot}48^2}{8 \times 18{,}000 \times 9} = 0{\cdot}0152 \text{ sq. in.} \quad A_{ta} \text{ is similar if } t = t_c.$$

In this method z should not exceed n or $D - n$, otherwise A_1 or N_1 will be outside the section. The additional steel required is usually comparatively small.

If z is such that A_1 is outside the section then N_1A_2 must be considered as in *Fig.* 4, in which N_1A_2DCB equals the area of the original stress-block NACB, and the moment of N_1A_2DCB about YY equals M_y. It follows that the area of the triangle EA_2N_1 must equal the area of the rectangle EDAN, and that the moment about YY of the triangle EA_2N_1 considered as a stress-block must also equal M_y.

If $EA_2 = y$, $EN_1 = x$, $DA = q$, and $AC = n$, then, since area EA_2N_1 = area EDAN, $\frac{x.y}{2} = b.q$; and since the moment of EA_2N_1 about YY = M_y,

$$\frac{3c}{4}.b.q.\left(\frac{b}{2} - \frac{y}{3}\right) = M_y.$$

Therefore, by substituting for y, $x = \dfrac{q}{\frac{3}{4} - \frac{2M_y}{cb^2q}}$.

Also the resistance of the concrete to M_x is reduced; the reduction in the moment of resistance is equal to the moment of the triangle EA_2N_1, considered as a stress-block, about the centroid of the area EDAN. Additional tensile and compressive reinforcement is required to provide a couple equal to this reduction.

Therefore $$\frac{3c}{4}.b.q.\left(\frac{x}{3} - \frac{q}{2}\right) = A_{ta}.t.s = A_{ca}.t_c.s,$$

in which s is the distance between A_t and A_c.

Hence $$A_{ta}.t.s = cbq\left(\frac{x}{4} - \frac{3q}{8}\right) \quad . \quad . \quad . \quad . \quad (9)$$

The additional tensile reinforcement can be placed only near corner E, which is in tension; the additional compressive reinforcement must therefore be placed on the same side of YY as E, that is near B, in order to avoid creating a moment about YY.

EXAMPLE V.—A section as in Example I, with P=70,000 lb., M_x=70,000 in.-lb., and n = 10·155 in. Calculate the additional reinforcement required if a moment M_y = 20,000 in.-lb. is imposed.

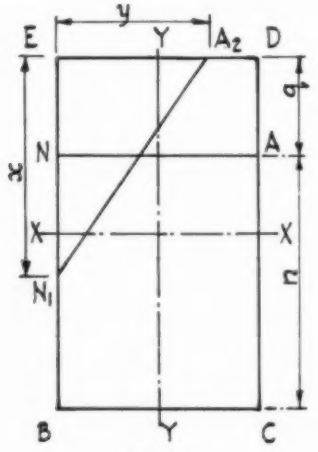


Fig. 4.

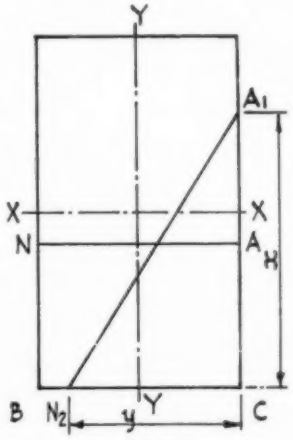


Fig. 5.

$q = 12 - 10{\cdot}155 = 1{\cdot}845$ in. $z = \dfrac{8 \times 20{,}000}{1000 \times 9^2} = 1{\cdot}98$ in., which exceeds q;

therefore $$x = \frac{1{\cdot}845}{0{\cdot}75 - \dfrac{2 \times 20{,}000}{1000 \times 9^2 \times 1{\cdot}845}} = 3{\cdot}83 \text{ in.}$$

$$A_{ta} \times 18{,}000 \times 9 = 1000 \times 9 \times 1{\cdot}845\left(\frac{3{\cdot}83}{4} - \frac{3 \times 1{\cdot}845}{8}\right).$$

Therefore $A_{ta} = 0{\cdot}0272$ sq. in., and, similarly, $A_{ca} = 0{\cdot}0272$ sq. in.

If NN_1 is such that N_1 is outside the section then N_2A_1 must be considered as in *Fig.* 5, in which the area N_2A_1C equals the area of the original stress-block NACB, that is $\frac{x.y}{2} = b.n$, and the moment of N_2A_1C about YY equals M_y.

Then $\frac{3c}{4}.n.b.\left(\frac{b}{2} - \frac{y}{3}\right) = M_y$, and, by substituting for y, $x = \dfrac{n}{\frac{3}{4} - \dfrac{2M_y}{cnb^2}}$.

The resistance of the concrete to M_x is again reduced by the moment of the triangle N_2A_1C as a stress-block about the centroid of area NACB. Additional tensile and compressive reinforcement is required to provide a couple equal to this reduction.

Therefore $$\frac{3c}{4}.b.n.\left(\frac{x}{3} - \frac{n}{2}\right) = A_{ta}.t.s = A_{ca}.t_c.s.$$

Hence $$A_{ta}.t.s = cbn\left(\frac{x}{4} - \frac{3n}{8}\right) \quad . \quad . \quad . \quad . \quad (10)$$

The additional compressive reinforcement can be placed only near the corner C which is in compression; the additional tensile reinforcement must therefore be placed on the same side of YY as the additional compressive reinforcement, that is near D, in order to avoid creating a moment about YY.

EXAMPLE VI.—In Example IV, if $M_y = 50{,}000$ in.-lb., calculate the additional reinforcement required. $z = \dfrac{8 \times 50{,}000}{1000 \times 9^2} = 4{\cdot}94$ in. This exceeds n, which equals 3·725 in.; therefore $x = \dfrac{3{\cdot}725}{0{\cdot}75 - \dfrac{2 \times 50{,}000}{1000 \times 3{\cdot}725 \times 9^2}} = 8{\cdot}92$ in.

$$A_{ca} \times 18{,}000 \times 9 = 1000 \times 9 \times 3{\cdot}725 \times \left(\frac{8{\cdot}92}{4} - \frac{3{\cdot}725 \times 3}{8}\right);$$

therefore $A_{ca} = A_{ta} = 0{\cdot}172$ sq. in.

A check on the compatibility of stresses and strains should be made in a manner similar to that explained in the previous article; accordingly, t_c and t should not exceed $50{,}000\left(1 - \frac{d_{c_1}}{n_1}\right)$ and $50{,}000\left(\frac{d_1}{n_1} - 1\right)$ respectively for mild steel, in which d_{c_1}, d_1 and n_1 are shown in *Fig.* 6. It should be noted that when n_1 is equal to or less than $0{\cdot}735d_1$, t for mild steel may be assumed to be 20,000 lb. per square inch in accordance with B.S. Code of Practice No. 114.

Suitable limitations of stress may be calculated for types of steel other than mild steel according to the relationship between their respective stress-strain characteristics at yield point and those of concrete.

If it is desired that all the steel in the section should reach its yield point when the member fails the value of z must be limited if there is any steel in the corner nearest to A_1 $\left(\text{if } n \text{ exceeds } \frac{D}{2}\right)$ or nearest to N_1 $\left(\text{if } n \text{ is less than } \frac{D}{2}\right)$.

A diagram showing the variation of the stress in the steel for the limiting condition when the distance between N_1A_1 and the nearest tensile reinforcement is a minimum consistent with the development of the full yield-stress at failure is shown in *Fig.* 7, Y_1Y_1 being the base-line. From the trigonometry of the diagram, $d_1 = d\cos\alpha + d_{ty}\sin\alpha$, $n_1 = n\cos\alpha + \frac{b}{2}\sin\alpha$, $\tan\alpha = \frac{2z}{b}$, and $n_1 = 0{\cdot}735\,d_1$, in which α is the angle between N_1A_1 and NA, and d_{ty} is the depth of side embedment of the tensile reinforcement.

From these relationships, $z = \frac{(0{\cdot}735d - n)b}{b - 1{\cdot}47d_{ty}}$. This represents the greatest value of z which will permit all the steel in the section to be used to its full capacity. If M_y exceeds $\frac{zcb^2}{8}$ (using the limiting value for z) the excess moment may be resisted by decreasing the area of steel near A_1 and increasing the area of steel on the other side of YY by the same amount to give a couple equal to the excess moment.

EXAMPLE VII.—A section as in Example I, with P=25,000 lb., M_x=200,000 in.-lb., $n = 6{\cdot}3$ in., and $A_t = 0{\cdot}486$ sq. in. in each corner. Calculate the additional reinforcement, if any, required to resist an additional moment $M_y = 30{,}000$ in.-lb.

If $d_{ty} = 1{\cdot}5$ in., max. $z = \frac{(0{\cdot}735 \times 10{\cdot}5 - 6{\cdot}3)9}{9 - 1{\cdot}47 \times 1{\cdot}5} = 1{\cdot}75$ in.

$$\frac{zcb^2}{8} = \frac{1{\cdot}75 \times 1000 \times 9^2}{8} = 17{,}700 \text{ in.-lb.}$$

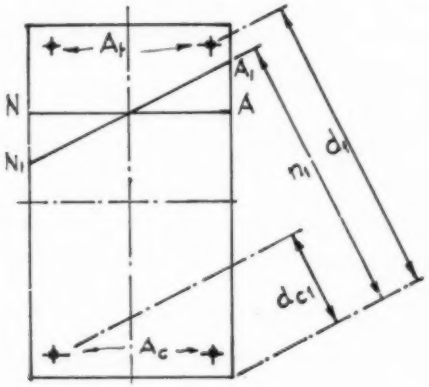


Fig. 6.

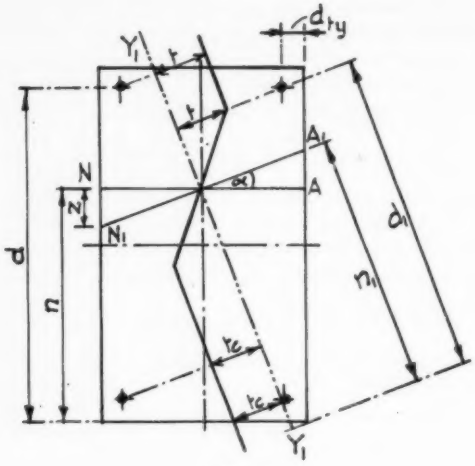


Fig. 7.

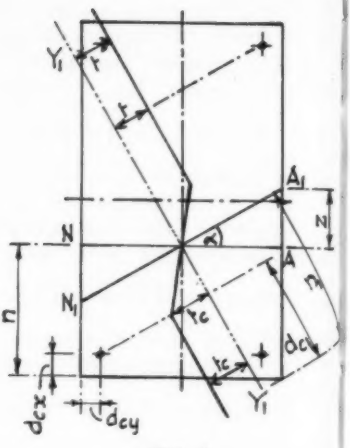


Fig. 8.

Excess moment to be resisted by steel = 30,000 − 17,700 = 12,300 in.-lb.

Adjustment to $A_t = \pm \dfrac{12{,}300}{18{,}000 \times (9 - 1{\cdot}5 - 1{\cdot}5)} = 0{\cdot}114$ sq. in.

Reinforcement near A_1, 0·486 − 0·114 = 0·372 sq. in. Reinforcement in farther corner, 0·486 + 0·114 = 0·600 sq. in. This reinforcement must also be adjusted as in Example IV to allow for the reduced moment of resistance of the concrete relative to XX due to the rotation of the neutral axis to N_1A_1.

If n is less than half D and the design for P and M_x includes compressive reinforcement, a limit must be placed on the value of z if the compressive reinforcement nearest to N_1 is to reach its yield-point when the member fails.

The variation of stress in the steel for the limiting condition when the distance between N_1A_1 and the nearest compressive reinforcement is a minimum consistent with the development of the full yield-stress at failure is as shown in *Fig.* 8, Y_1Y_1 being the base-line. From the trigonometry of the diagram,

$$d_{c1} = d_{cx}\cos\alpha + (b - d_{cy})\sin\alpha,\quad n_1 = n\cos\alpha + \frac{b}{2}\sin\alpha,\quad \tan\alpha = \frac{2z}{b}.$$

Since $$t_c = 50{,}000\left(1 - \frac{d_{cx}}{n}\right),\quad 18{,}000 = 50{,}000\left(1 - \frac{d_{cx}}{n}\right).$$

Therefore $d_{cx} = 0{\cdot}64n_1$, in which α is the angle between N_1A_1 and NA, and d_{cy} is the depth of side embedment of the compressive reinforcement.

From these relationships, $z = \dfrac{(0{\cdot}64n - d_{cx})b}{2(b - d_{cy}) - 0{\cdot}64b}$. This represents the greatest value for z in order that all the steel in the section may be used to its full capacity. If M_y exceeds $\dfrac{zcb^2}{8}$ (using the limiting value for z) the excess moment is resisted in a manner similar to that described for Example VII.

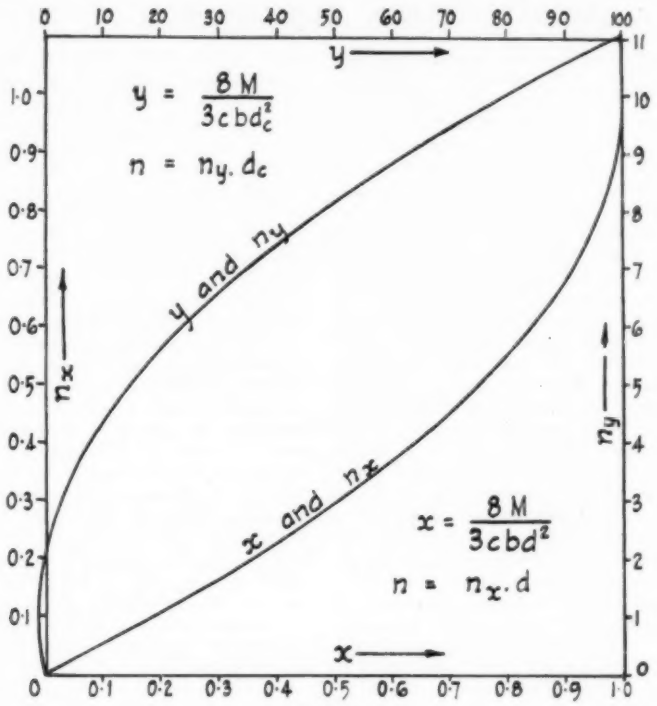


Fig. 9.

Should the value of n be appreciably less than half D the design may be restricted initially to tensile reinforcement only, and so avoid consideration of the stress in the steel at the corner nearest to N_1. A small amount would be needed in the other corner when the adjustment is made to allow for the reduction in the moment of resistance of the concrete.

The foregoing methods give different amounts of steel at each corner of a rectangular section. However, a member will not be weakened if there is additional steel at one or more corners in order to obtain a symmetrical arrangement or to allow for more than one condition of loading. It should also be realised that although these methods give reasonable results they are not exact, being based largely on an idealised representation of the stresses and strains at failure.

The graphs in *Fig.* 9 enable the depth of the neutral axis for P and M_x only (in accordance with formulæ (5) and (8) in the previous article) to be quickly determined.

Book Reviews.

"Guide to the R.I.B.A. Forms of Contract." By Donald Keating. (London: Sweet & Maxwell, Ltd. 1959. Price 30s.)

THE fact that so much of the time of the Courts is occupied in attempting to find the meaning of Acts of Parliament is an indication of the difficulty of preparing a document in such a way that there can be no doubt about the meaning of every sentence and every word. Forms of contract used in the building industry are no exception, as many architects, engineers, and builders have discovered to their cost. In this book the Forms of Contract (which include what are commonly known as the conditions of contract) issued by the Royal Institute of British Architects are printed in full, and each clause is followed by the author's interpretation of its meaning. The value of the book is increased by the inclusion of many decisions of the Courts, which have produced a mass of "case law" to which reference may be made. It may be pointed out that one must be careful in basing an opinion on case law, because of the differences in the reasons for most disputes. Also there are no doubt further subjects for argument that have not yet come before the Courts. R.I.B.A. Forms of Contract have been in use for more than fifty years, but in this book a report is given of a case heard in the Court of Appeal only a few months ago on whether the architect or the builder was bound by this form of contract to pay the costs incurred as a result of an infringement of a local by-law. However, a book such as this, written by a barrister-at-law, is a useful work of reference to a layman.

"Structural Design for Dynamic Loads." By C. H. Norris, R. J. Hansen, M. J. Holley, Jr., J. M. Biggs, S. Namyet, and J. K. Minami. (London: McGraw-Hill Publishing Co., Ltd. 1959. Price 97s.)

THE dynamic effects dealt with in this book, which is based mainly on lectures given at the Massachusetts Institute of Technology, include blast from explosions, movement due to earthquakes, and, less comprehensively, vibration due to traffic and the effects of wind. The behaviour of materials subjected to such dynamic effects is considered first, and the section dealing with concrete is noteworthy for its conciseness and value. The bond stress between concrete and steel recommended when loads are applied rapidly is 15 per cent. of the crushing strength of concrete cylinders, which is 50 per cent. greater than for static loads. This comparatively small increase is recommended because of the lack of data regarding members subjected to dynamic loads and because of the serious results arising from failure of bond. The theory of the effect on a structure of a dynamic load is followed by a simplified analysis for use in design. Methods of making the more complex calculations by numerical integration and by computors are described.

Consideration of resistance to blast includes the effect of atomic explosions as well as more common causes. The chapters on earthquakes, to which a professor of Waseda University, Japan, makes a contribution, are interesting since the codes for earthquake-resistant buildings in the United States and Japan are reviewed, and concise data are given for the design of such structures in steel or reinforced concrete. The Japanese method of composite steel and concrete construction is not described.

"Practical Problems in Soil Mechanics." By H. R. Reynolds and P. Protopapdakis. (London: Crosby Lockwood & Son, Ltd. Third edition. 1959. Price 21s.)

THE additional matter in this edition includes more information on piled foundations and brief particulars of sand piles for roads laid on weak ground, the effect of vibration on soil, and the determination of the properties of soils by means of nuclear radiation. Since it is stated that research on these matters is proceeding, the information now given may not be final.

"Das Beton-ABC." By Alfred Hummel. (Berlin: Wilhelm Ernst & Sohn. 1959. Price D.M. 21.60.)

THE twelfth edition of this book on making concrete is extensively revised and enlarged and, although the practical manner of dealing with the subject is retained, the scientific basis receives more attention. Most of the well-known methods of improving the quality of concrete are described.

Prestressed Hollow Bridge Beams.

BRIDGES with spans up to 295 ft. long have been built in Cuba with hollow beams prestressed with galvanised strands that are not embedded in concrete and that can be inspected by men walking inside the beams. These bridges have been described in "Engineering News-Record" in an article by Mr. H. K. Preston, who gives some notes on the design of such bridges.

As an example, the writer refers to the design shown in *Fig.* 1. Such a bridge would comprise three longitudinal cells continuous over three spans of 152 ft. 6 in. each. It would have two carriageways each 12 ft. wide on the top flange and two

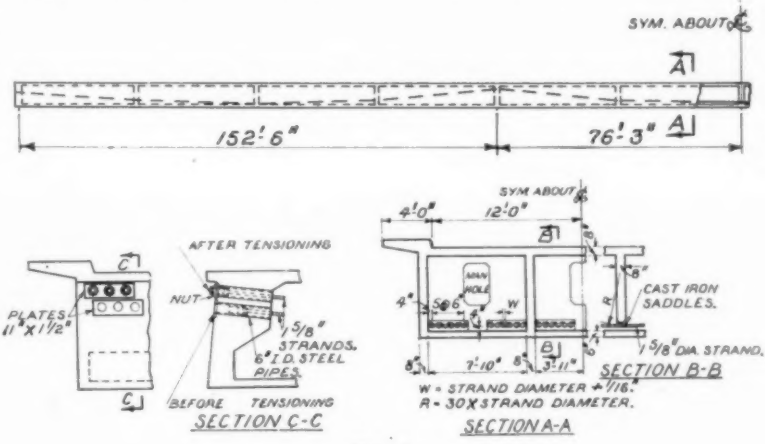


Fig. 1.

4-ft. cantilevered footpaths. The concrete would have a minimum ultimate strength of 4000 lb. per square inch and the initial prestress in the strands at the centre of the bridge would be assumed to be 115,000 lb. per square inch. No tension would be permitted in the concrete due to bending stresses under 1·2 times the dead load plus twice the live load and impact.

The depth of the beam at the piers would be between 4 per cent. and 5 per cent. of the span. The prestressing strands should be placed as high as possible over the piers and as low as possible at points of maximum positive bending moments. The centre of a strand of $1\frac{1}{16}$ in. diameter can be at 4 in. from the inside surface of the top or bottom slab (this clearance is needed for the end fittings on the strands).

Since the dead weight creates a large portion of the total moment, it is desirable to place the piers so that they produce a large negative moment on the beam above them. In one of the Cuban bridges, for example, short cantilevers at both ends of the main span develop the negative moments at the piers. However, three equal spans provide a more economical design for continuous multiple-span structures. Transverse walls are usually placed at the quarter-points and the centre of each span, but they are seldom less than 35 ft. or more than 50 ft. apart. The rise at the quarter-points should be one quarter of the rise of the strand between midspan and the pier.

After the superstructure is built the strands are installed. The reel containing the strands is mounted on a shaft at one end of the bridge; the strands are threaded through pipes in the ends of the beams and at the transverse walls they are held in position by cast-iron saddles.

Each strand is tensioned from both ends simultaneously to reduce frictional losses. Three or more days after initial tensioning the strands are again tensioned

to compensate for losses due to the creep of the steel and shrinkage of the concrete. Between the greased galvanised strands and the cast-iron saddles friction is so small that it is not necessary to use a hinge. After installation, all the strands are cleaned and painted.

The following design procedure is suggested. (1) Assume a final tension F in the strand of 1,000,000 lb.; (2) compute the vertical forces due to F that act at each transverse wall; (3) compute the stresses due to these vertical loads; (4) compute the direct compressive stress $\frac{F}{A}$ and add it algebraically to the stresses obtained from (3). The result is the net stresses due to F. (A is the cross-sectional area of the beam); (5) calculate the stresses due to dead and live loads and ensure that no tension will occur in the concrete under specified combinations of these loads; (6) divide these stresses (for critical sections) by the net stresses due to F and multiply by 1,000,000 to obtain the net prestressing force required. From this, the number of strands needed can be calculated.

Since the strands must resist some of the shearing force, the forces on the piers of a three-span bridge are not the same as for an unprestressed structure with the same elastic properties. The forces can be determined as follows: (1) Compute the forces on the piers due to dead load as for an unstressed structure having the same elastic properties. (2) Compute the forces on the piers due to the upward forces of the strands at the transverse walls in the beams. (3) Compute the downward forces of the strands over the piers. (4) Add the forces calculated in (1), (2), and (3) to obtain the total forces acting on the piers. (5) The effect of live-loads will be the same as for an unstressed structure with the same elastic properties.

Parking Garages.

SOME notable parking garages built abroad in reinforced concrete are illustrated and briefly described in a brochure of 16 pages issued by the Cement & Concrete Association. Some of the garages are of novel and interesting design (see facing page).

Fig. 1 shows a garage built at Düsseldorf, Germany, in 1953. It is four stories high and measures 328 ft. long by 184 ft. wide. Entry and exit ramps are suspended along the main façades from cantilevered projections of the roof beams. The ramps have a slope of 1 in 7 with level landings at each floor level. Service and repair shops are on the ground floor, and the upper floors provide accommodation for 600 cars. The structure is formed of two rows of rectangular frames with expansion joints between the adjoining columns at the centre. The floors are of prestressed concrete. The garage is operated by only six men and four office staff.

The garage shown in *Fig.* 2 is at Salt Lake City, Utah, U.S.A. Accommodation is available on five levels for 550 cars parked at an angle of 60 deg. The building is supported on forty prestressed precast columns 52 ft. high, and was built in ten parts each 60 ft. square. The four columns for each section were erected to roof level by two cranes and were braced to form a tower. Concrete for the roof slab was then placed. After the concrete in the roof had matured winches were placed on the slab and the shuttering was lowered to the next level, so that the floors were built from the top downwards; it is stated that this was cheaper than concreting the lowest floor first and raising the shutters for the upper floors.

Copies of the brochure, which is entitled "Structural Concrete Parking Garages", are obtainable (free) on application to the Association at 52 Grosvenor Gardens, London, S.W.1.

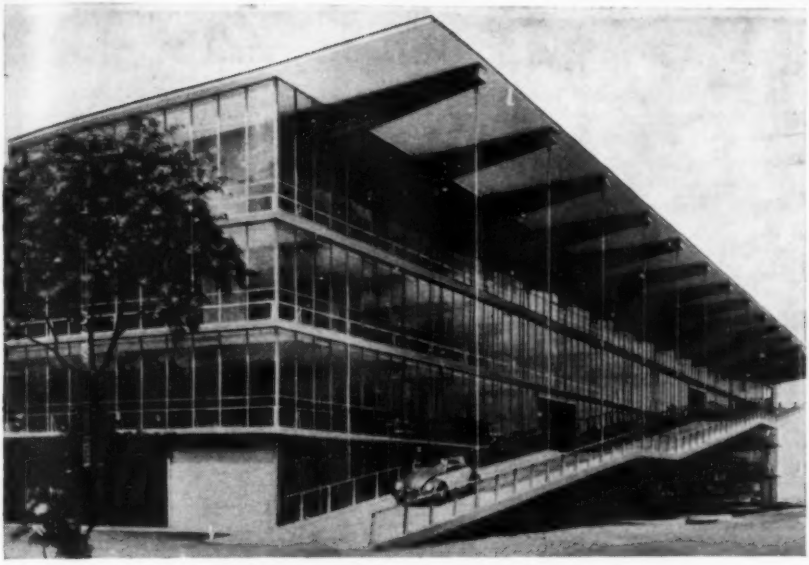


Fig. 1.—Parking Garage at Düsseldorf, Germany.

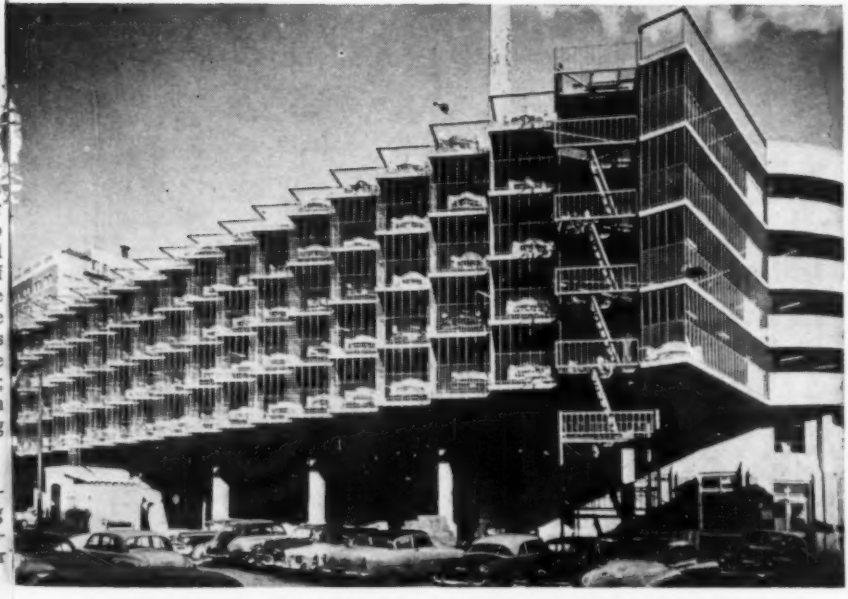


Fig. 2.—Parking Garage at Salt Lake City, U.S.A.

(*See facing page.*)

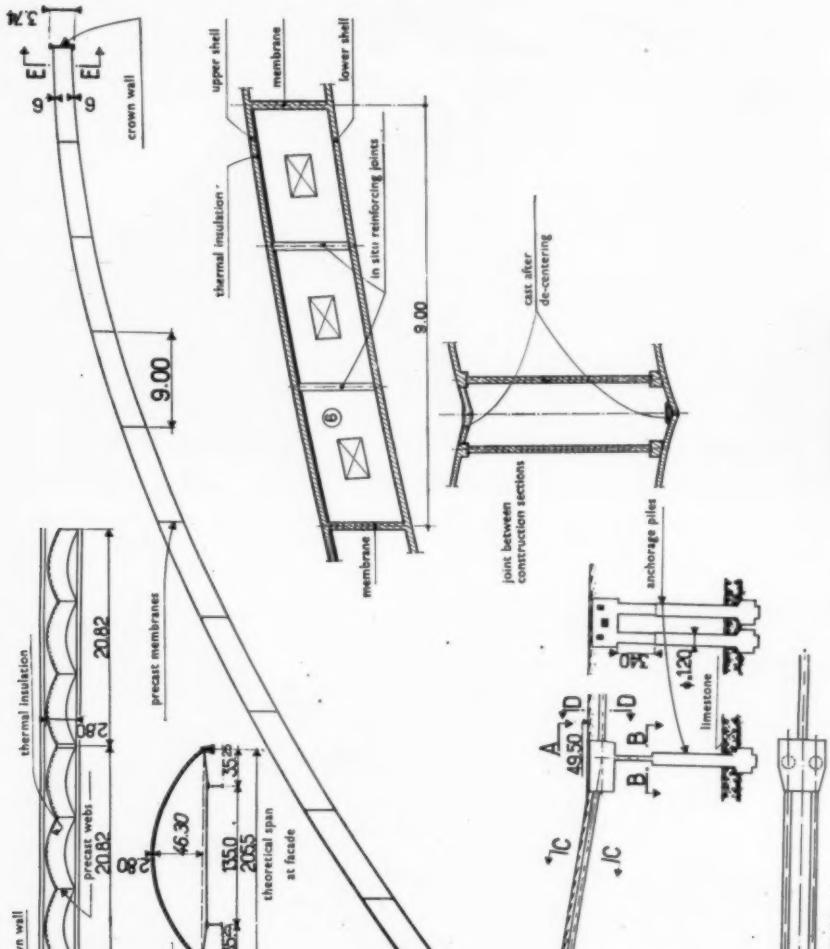


Fig. 2.—Shell Roof in Paris: Details of Roof and Ties.

A Large Shell Roof in Paris.

THE roof of the exhibition hall (*Fig.* 1) of the National Centre of Industries and Technology, Paris, covers an area of 226,000 sq. ft. It is an equilateral triangle on plan, and is supported at three points only (*Fig.* 2). The distance between the supports is 676 ft. and it is claimed to be the largest span of any shell roof in the world. The abutment and foundation at each support is a massive vertical structure 20 ft. high bearing on good ground.

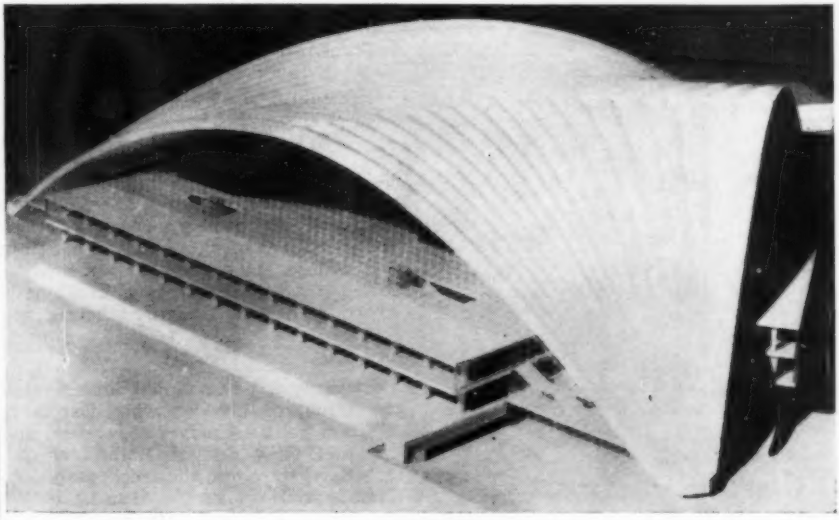


Fig. 1.

The abutments are connected by underground prestressed ties, details of which are shown in *Fig.* 2 (see facing page).

The horizontal distance from the centre of a support to the crown of the roof is 390 ft., and the equivalent span as an arch is 780 ft. The roof is a double shell similar in appearance to a groined vault without ribs, and is corrugated, each corrugation being a separate sector. There are nine sectors in each of the six triangular areas meeting at each crown-beam or "wall" (*Fig.* 2). The theoretical half-spans of the sectors vary from 338 ft. at the façade to 390 ft. at the groin. The theoretical rise is 152 ft. The total height of each sector, measured perpendicularly to the centroidal axis, is 6·2 ft. at the crown and 9 ft. at the springing. The projected width of each sector is 22 ft. at the crown and 1 ft. 10 in. at a distance of 43 ft. from the springing. The radii of curvature of the intrados of the corrugations vary from 22 ft. 7 in. at the crown to 10 ft. 10 in. at a distance of 43 ft. from the springing. The change of section provides a constant factor of safety against local buckling. The radii of curvature in a vertical plane at right-angles to the façades are identical, thus permitting the re-use of shuttering.

The thickness of each of the two shells between the crown and a section 98 ft. from the springing is about $2\frac{1}{2}$ in., and increases to about $23\frac{1}{2}$ in. at the abutment. The two shells are connected by vertical precast reinforced concrete longitudinal webs $2\frac{3}{4}$ in. thick (*Fig.* 5), which provide resistance to shearing and enable the two shells to act together. The web at the crown is cast in place and is 3 in. thick. Double webs are provided longitudinally at the temporary joints between each group of three sectors.

The façades and the shells deform independently. Vertical precast transverse webs (called "membranes" in

Fig. 2.—Shell Roof in Paris: Details of Roof and Tie.

PLAN SECTION AA

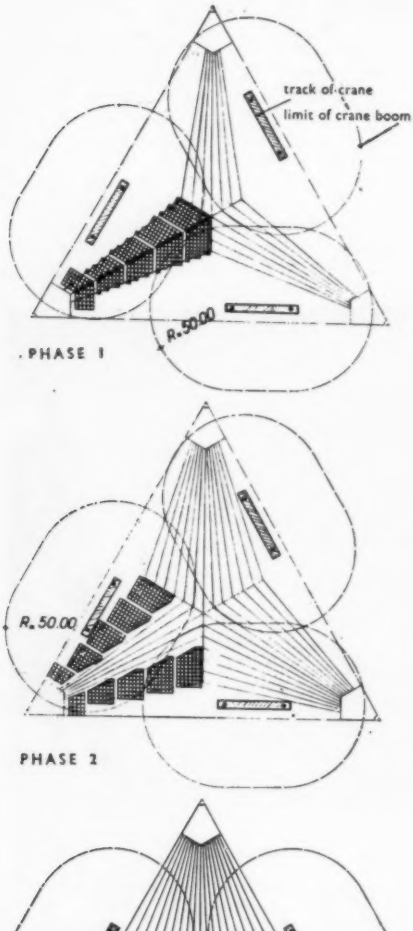


Fig. 3.—Stages of Construction of Roof.

Fig. 2) 2¾ in. thick are provided at intervals of 30 ft. at right-angles to the façades in order to transmit wind loads, distribute the concentrated unsymmetrical forces on the shells, and act as frames between the shells. The junctions of the webs and shells are of concrete cast in place and are designed to prevent rupture due to shearing.

Openings in the webs provide means of ventilation and equalise the temperature of the shells. The underside of the upper shell is lined with 1½-in. cement-impregnated fibreboard to provide thermal insulation and act as permanent shuttering. The openings in the webs permitted the transfer of shoring from one part of the roof to another during construction. Water-tightness is ensured by using a dense concrete for the shells and further protection is obtained by polyester paint applied to the top of the upper shell.

The crown-beams are of I-section 12 ft. deep and comprise a web 11 in. thick and flanges 15 in. thick and from 2 ft. to 4 ft. wide. The beams ensure monolithic action of the sectors meeting at each beam where the deformations are greatest.

Design.

Owing to the exceptional dimensions of the structure the following factors in addition to the loads were taken into account in the design: Method of construction; errors in the thickness of the shell; variations in the density of the concrete; deviations between the theoretical and the actual points of application of forces due to the heterogeneity of the structure and to possible constructional errors affecting the centre-line of the roof or the axes of its components; differences of temperature, shrinking and elastic modulus of the two shells as well as shrinking, creep, and change of temperature of the entire roof; second-degree deformations; compensation for errors during the removal of the centering under the arch. The effects of elastic contraction, creep, shrinking, and changes of temperature were compensated for largely by removing the centering with the aid of jacks which acted parallel to the centroidal axis and were inserted in gaps 3 ft. 3 in. wide formed about 36 ft. from the centre of each abutment. At this position the shell is about 8 in. thick and could resist

the thrust from the jacks without extra reinforcement.

Two features of the roof are (a) the average thickness of concrete is constant along each horizontal generator in the planes at right-angles to the façades, and (b) the centroidal axis of each façade is a catenary corresponding to the variation of the dead load of the arch adjacent to the façade. As the variable height of the webs and the presence of the crown-beam made it impracticable to consider each sector as complying exactly with this curve, a compromise was adopted, namely, a curve of the fourth degree valid for all sectors.

Compressive stresses at the top of the shell due to dead load vary from 434 lb. per square inch at the crown to 1152 lb. per square inch at the springing.

The live loads comprise a uniformly-distributed load of 10 lb. per square foot without wind, or 5 lb. per square foot acting in the least favourable manner with the wind; a service load of 3 lb. per square foot, distributed in the least favourable manner; a pressure due to a wind of 75 miles per hour. Wind-tunnel tests of a model demonstrated that the most adverse cases are when the wind blows at right-angles to a façade, with three façades in place, or when this façade is destroyed and the other two are intact. An allowance of 2½ lb. per square foot in the least favourable position was assumed for possible errors in the thickness of the shells. A temperature variation of plus or minus 20 deg. C. with a modulus of elasticity of 4,300,000 lb. per square inch was assumed. Due to the live loads, the compressive stresses at the underside are about 2060 lb. per square inch at the springing, 1660 lb. per square inch at one-fifth of the half-span, and 1000 lb. per square inch at the crown. Tensile stresses of 88 lb. per square inch at the crown occur if the wind blows through a destroyed façade. The façades are of glass.

Calculation of the resistance to buckling of the roof in general was based on Euler's formula, the elastic modulus being

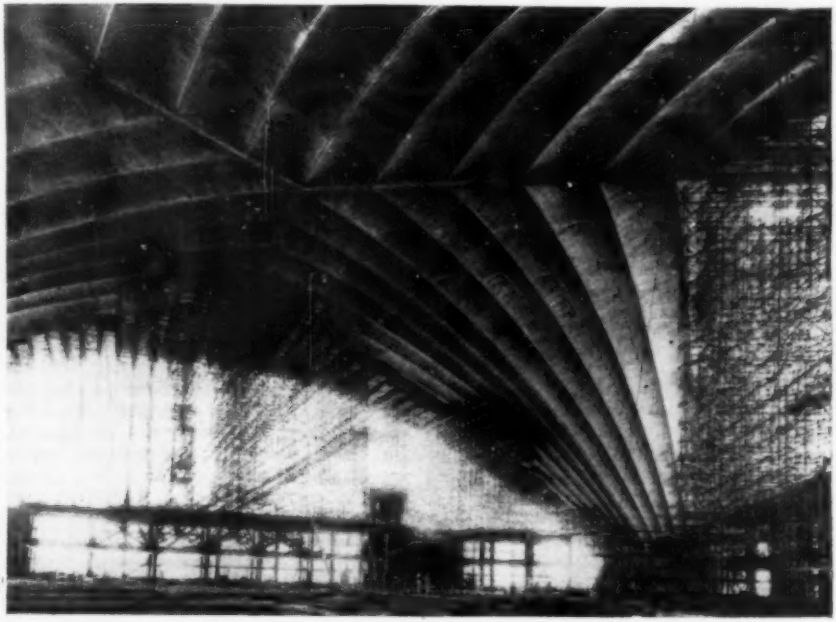


Fig. 4.

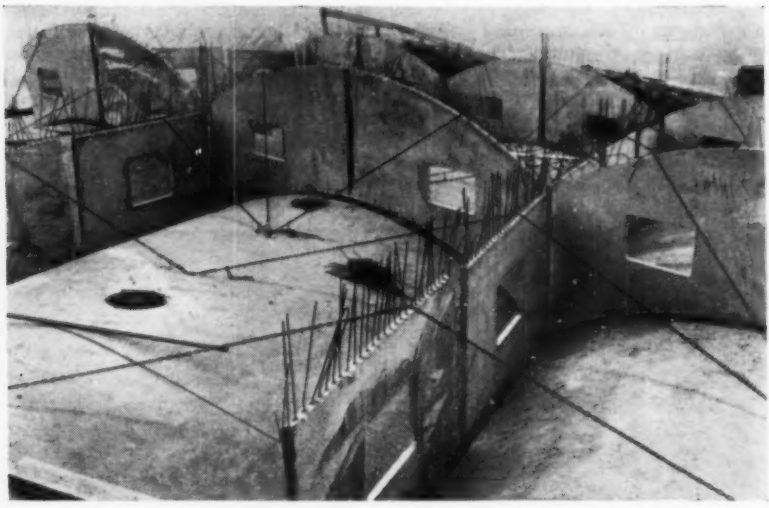


Fig. 5.—Vertical Webs between Vaults.

assumed to be 1,420,000 lb. per square inch in the calculation of the creep so as to obtain a factor of safety greater than four. Since it is unlikely that arches under constant thrust fail abruptly due to buckling, it was thought to be more accurate to anticipate an increase in actual deformation, considering the maximum deviation of the neutral plane under the effects of possible structural errors and of eccentricity of forces caused by unsymmetrical live loads and resulting second-degree deformations. Such calculations indicated that the deformed roof has a factor of safety of not less than two.

Construction.

The roof was constructed in three main phases, each of which comprised eighteen sectors as shown in *Fig.* 3. A stiff concrete, which included a plasticiser and was compacted by vibration, was used to ensure that there would be no displacement of the concrete when it was placed at a maximum angle of 35 deg. without top shuttering. The centering for the first stage was used for each succeeding stage. The shuttering was dismantled, but the scaffolding was transferred almost intact on rail-tracks to the next position. A joint was left between adjoining sectors cast in successive stages to allow for differential deflection while removing the centering. After several months, when the greater part of the creep and shrinking of the concrete had taken place, the joints were filled with concrete cast in place to form a monolithic structure. A view of the third stage of construction is shown in *Fig.* 4.

The notes in the foregoing are taken from a paper entitled " The Design and Construction of the Shell Roof of the Exhibition Palace of the National Centre of Industries and Technology, Paris ", read by M. Nicolas Esquillan at a meeting of the Société des Ingénieurs Civils de France in London in December, 1958.

Roof of Stadium Supported by Cables.

PRESTRESSING STEEL TENSIONED BY LOADING.

THE City of Montevideo recently invited tenders for the design and construction of a stadium. The only requirements specified were that the structure should be permanent and suitable for use by day and night, that the first stage of construction should require only the roof and its supporting elements, that the diameter of the arena be not less than 135 ft., and that accommodation be provided for not fewer than 20,000 persons. Intermediate supports were prohibited, and the time permitted for completion of the roof was fourteen months.

The design accepted (*Fig.* 1) consists of a reinforced concrete wall 310 ft. in diameter and 85 ft. high (*Fig.* 2) and a roof supported by 256 7-strand prestressing cables extending from a beam on top of the wall to a steel ring of 18 ft. internal diameter over the arena. The wall is 4 in. thick and supports a vertical load of 850 tons in addition to the force of the wind; at intervals of 15 ft. around its circumference it is supported by pairs of piles. The wall was cast in moving forms 2 ft. 4 in. deep. Two sets of forms were used, and one ring of forms was raised every day; internal and external vibrators were used.

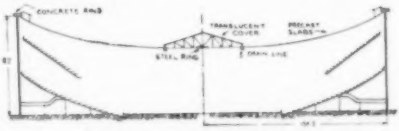


Fig. 1.—Cross Section.

The ring (*Figs.* 1 and 3) supporting the glass roof over the arena is composed of welded steel sections and was fabricated from two parallel 2-in. by 12-in. plates held 2 in. apart by a continuous plate on the inside face of the ring. The ring is glazed to supply natural lighting, and the remainder of the roof is covered by precast concrete slabs.

The horizontal forces due to the roof are resisted by the beam on the wall; this beam is 6 ft. 7 in. wide and 1 ft. 6 in. deep, and is secured to the wall by dowel bars. When the beam had the desired strength the steel ring was supported on scaffolding at a higher level than that required in order to facilitate the placing of the prestressing strands. When all the strands were in position the scaffolding was removed and the steel ring lowered to the

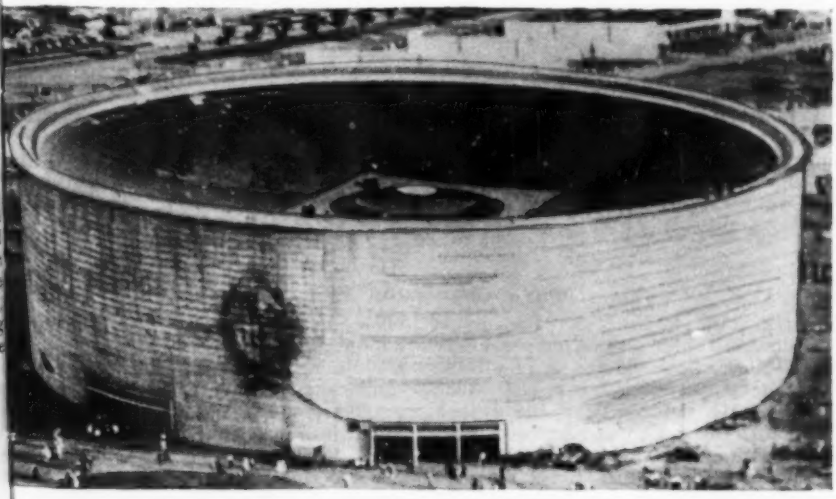


Fig. 2.—Completed Structure, showing Dished Roof.

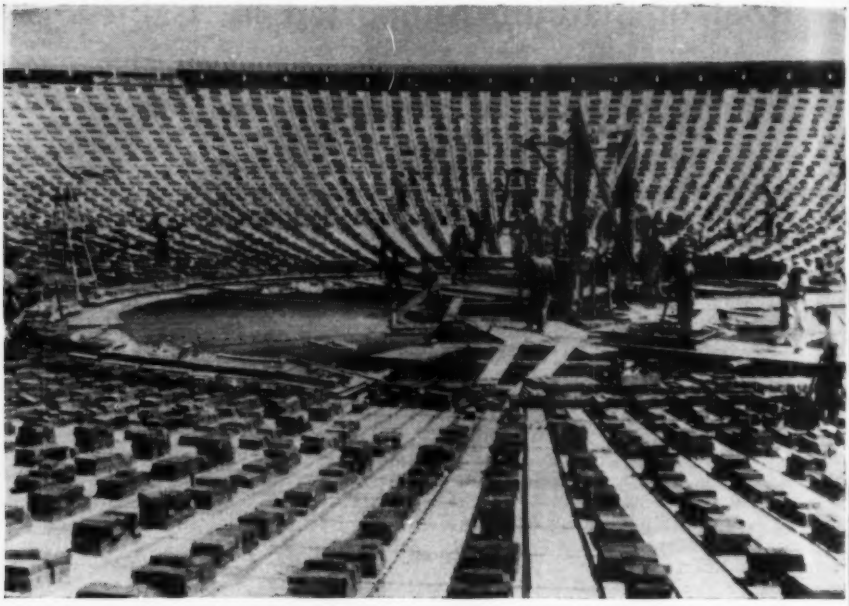


Fig. 3.—Roof Loaded to Tension Prestressing Steel.

required height. The strands were then adjusted at the anchorages to obtain uniform support of the roof.

Placing of the precast slabs was commenced at the steel ring and progressed outwards, the previously placed slabs serving as a working platform. The slabs are 3 ft. 3 in. long and of variable width according to their position on the roof. Hooks were incorporated in the slabs to engage with the radial steel strands.

When all the slabs were in position the roof was loaded by placing bricks over the whole of its area (*Fig.* 3). This load stretched the cables so that gaps appeared between the slabs. Asbestos-cement boards were then wired in place beneath the joints and the gaps filled with cement mortar. When this mortar had gained sufficient strength the loading was removed and the roof left in a permanent state of compression.

The structure was designed by L. I. Viera, L. A. Mondino, and A. S. Miller (of Montevideo) and was constructed by Mondino y Viera, Ltd. The Preload Co. Inc., were consultants for the prestressing work.

Metric Conversion Factors.

BRITISH Standard No. 350, "Conversion Factors and Tables", which has previously been issued as one volume, is now being issued in two parts. The first part, which was recently published by the British Standards Institution at a price of 15*s*., describes the British and metric units of measurement used in connection with metrology, mechanics, heat, and associated branches of physics. Fifty tables are given relating similar units one with another, but reference tables from which the number of British units of one kind can be readily converted to the equivalent number of metric units (or vice versa) are to be given in Part 2. The minute differences between the U.S.A. and British units of length and weight are given, as well as the more significant differences in volumetric measures.

Design of Helical Staircases—4.*

Statically-Indeterminate Cases.

By JACQUES S. COHEN.

Circular Helical Beam with One End Fixed and One End Pin-Jointed (Fig. 16).

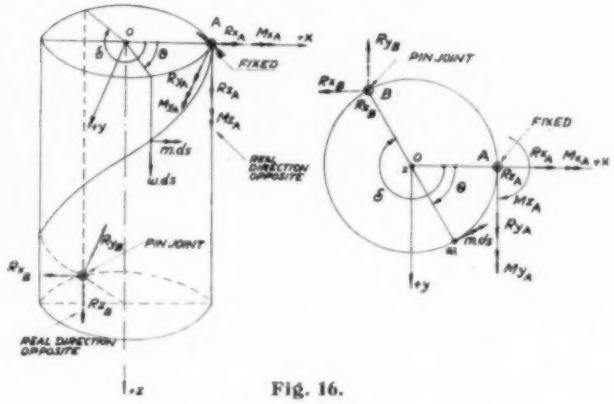


Fig. 16.

WRITING $\theta = 0$ and $\theta = \delta$ in equations (45), (47), and (24), and inserting these in conditions (49), the conditions of restraint are given by (59).

Values of C_7 to C_{12} are obtained respectively from equations 59 (a), (b), (c), (d), (f) and (h) and inserted in the remainder to obtain equations (60).

In equations (60) there are only six unknown factors, as A_1 to A_6 and B_1 to B_6 are obtained from equations (44) and (46) and the values of M_{t_A} and M_{n_A} for $\theta = 0$ are as follows [from equations (24)]:

$$M_{t_A} = C_4 + C_6; \quad M_{n_A} = \frac{1}{\sin\phi}(C_5 + C_2 a \cot\phi + wa^2 + ma).$$

Substituting these values in equations (60), multiplying the first three equations by $\dfrac{\sin^2\phi}{K_1 a^2 \cos\phi} \times \dfrac{\sin\phi\cos\phi}{a}$, and rearranging, equations (61) with "unknowns" C_1 to C_6 are obtained.

In equations (61) ε and ε_1 to ε_4 are as previously defined.†

Equations (61) may be solved to obtain the values of the six constants for a helical beam fixed at one end and pin-jointed at the other, and the insertion of these in equation (24) gives the internal forces and moments at any point of the beam.

In the two cases previously considered, the symmetry of the load and the similarity of conditions at both supports make an arithmetical check possible by

* Concluded from August, 1959.

† See equation (53) this Journal for July, 1959.

$$
\begin{aligned}
&\text{(a)}\quad \psi_{M_A} = 0 = C_7 + C_9.\\
&\text{(b)}\quad \psi_{N_A} = 0 = C_8 + A_1 - \frac{A_3}{2} - \frac{3}{4}A_4 - \frac{K_1 a a}{\sin\phi} M_{t_A}.\\
&\text{(c)}\quad \psi_{D_A} = 0 = -C_9 - A_2 + A_3 + A_5 - K_1 a(1+\sigma) M_{n_A}.\\
&\text{(d)}\quad D_{t_A} = 0 = C_{10} + C_{12}.\\
&\text{(e)}\quad D_{t_B} = 0 = C_{10} + C_{11}\sin\delta + C_{12}\cos\delta + B_1\delta - B_2\frac{\delta\sin\delta}{2} - B_3\frac{\delta\cos\delta}{2}\\
&\qquad + B_4\Big(\frac{-\delta^3\sin\delta - 3\delta\cos\delta}{4}\Big) + B_5\Big(\frac{-\delta^3\cos\delta + 3\delta\sin\delta}{4}\Big)\\
&\qquad + B_6\frac{\delta^2}{2} + B_7\Big(\frac{-\delta^3\cos\delta}{6} + \frac{3\delta^2\sin\delta}{4} + \frac{7\delta\cos\delta}{4}\Big)\\
&\qquad + B_8\Big(\frac{-\delta^3\sin\delta}{6} - \frac{3\delta^2\cos\delta}{4} + \frac{7\delta\sin\delta}{4}\Big).\\
&\text{(f)}\quad D_{n_A} = 0 = C_{11} + B_1 - \frac{B_2}{2} - \frac{3}{4}B_4 + \frac{7}{4}B_7.\\
&\text{(g)}\quad D_{n_B} = 0 = C_{11}\cos\delta - C_{12}\sin\delta + B_1 - B_2\Big(\frac{\delta\cos\delta + \sin\delta}{2}\Big)\\
&\qquad - B_3\Big(\frac{-\delta\sin\delta + \cos\delta}{2}\Big) + B_4\Big(\frac{-\delta^3\cos\delta + \delta\sin\delta - 3\cos\delta}{4}\Big)\\
&\qquad + B_5\Big(\frac{\delta^3\sin\delta + \delta\cos\delta + 3\sin\delta}{4}\Big) + B_6\delta\\
&\qquad + B_7\Big(\frac{\delta^3\sin\delta}{6} + \frac{\delta^2\cos\delta}{4} - \frac{\delta\sin\delta}{4} + \frac{7}{4}\cos\delta\Big)\\
&\qquad + B_8\Big(\frac{-\delta^3\cos\delta}{6} + \frac{\delta^2\sin\delta}{4} + \frac{\delta\cos\delta}{4} + \frac{7}{4}\sin\delta\Big).\\
&\text{(h)}\quad D_{b_A} = 0 = -C_{12} - B_2 + B_5 + B_6 + 2B_8.\\
&\text{(i)}\quad D_{b_B} = 0 = -C_{11}\sin\delta - C_{12}\cos\delta - B_2\Big(\frac{-\delta\sin\delta + 2\cos\delta}{2}\Big)\\
&\qquad - B_3\Big(\frac{-\delta\cos\delta - 2\sin\delta}{2}\Big) + B_4\Big(\frac{\delta^3\sin\delta - \delta\cos\delta + 4\sin\delta}{4}\Big)\\
&\qquad + B_5\Big(\frac{\delta^3\cos\delta + \delta\sin\delta + 4\cos\delta}{4}\Big) + B_6\\
&\qquad + B_7\Big(\frac{\delta^3\cos\delta}{6} + \frac{\delta^2\sin\delta}{4} + \frac{\delta\cos\delta}{4} - 2\sin\delta\Big)\\
&\qquad + B_8\Big(\frac{\delta^3\sin\delta}{6} - \frac{\delta^2\cos\delta}{4} + \frac{\delta\sin\delta}{4} + 2\cos\delta\Big)\\
&\qquad - \frac{a}{\sin\phi\cos\phi}\Big[-C_9\sin\delta - C_9\cos\delta - A_2\Big(\frac{2\cos\delta - \delta\sin\delta}{2}\Big)\\
&\qquad - A_3\Big(\frac{-2\sin\delta - \delta\cos\delta}{2}\Big) + A_4\Big(\frac{\delta^3\sin\delta - \delta\cos\delta + 4\sin\delta}{4}\Big)\\
&\qquad + A_5\Big(\frac{\delta^3\cos\delta + \delta\sin\delta + 4\cos\delta}{4}\Big) + A_6\\
&\qquad + \sin^2\phi\Big\{C_7 + C_8\sin\delta + C_9\cos\delta + A_1\delta\\
&\qquad - A_2\frac{\delta\sin\delta}{2} - A_3\frac{\delta\cos\delta}{2} + A_4\Big(\frac{-\delta^3\sin\delta - 3\delta\cos\delta}{4}\Big)\\
&\qquad + A_5\Big(\frac{-\delta^3\cos\delta + 3\delta\sin\delta}{4}\Big) + A_6\frac{\delta^2}{2}\Big\}\Big].\\
&\text{(j)}\quad M_{t_B} = 0 = C + C_5\sin\delta + C_6\cos\delta + a\delta\cot\phi(C_2\cos\delta - C_3\sin\delta) + wa^2\delta.\\
&\text{(k)}\quad M_{n_B} = 0 = C_5\cos\delta - C_6\sin\delta\\
&\qquad + a\cot\phi\{C_2(\cos\delta - \delta\sin\delta) - C_3(\sin\delta + \delta\cos\delta)\} + wa^2 + ma.\\
&\text{(l)}\quad M_{b_B} = 0 = -\cot\phi(C_2\sin\delta + C_3\cos\delta) + \tan\phi C_4\\
&\qquad + \delta a\cot^2\phi(-C_2\cos\delta + C_3\sin\delta)\\
&\qquad - \frac{a}{\sin^2\phi}(C_2\sin\delta + C_3\cos\delta) - \frac{a}{\cos^2\phi}C_1 - \delta wa^2\cot\phi.
\end{aligned}
\tag{59}
$$

(a) $\left(-B_1+\frac{B_2}{2}+\frac{3}{4}B_4-\frac{7}{4}B_7\right)\frac{\sin\delta}{\delta}+(-B_2+B_5+B_6+2B_8)\frac{\cos\delta-1}{\delta}$
$+B_1-B_2\frac{\sin\delta}{2}-B_3\frac{\cos\delta}{2}+B_4\left(\frac{-\delta\sin\delta-3\cos\delta}{4}\right)$
$+B_5\left(\frac{-\delta\cos\delta+3\sin\delta}{4}\right)+B_6\frac{\delta}{2}+B_7\left(\frac{-\delta^2\cos\delta}{6}+\frac{3\delta\sin\delta}{4}+\frac{7}{4}\cos\delta\right)$
$+B_8\left(\frac{-\delta^2\sin\delta}{6}-\frac{3\delta\cos\delta}{4}+\frac{7\sin\delta}{4}\right)=0.$

(b) $\left(-B_1+\frac{B_2}{2}+\frac{3}{4}B_4-\frac{7}{4}B_7\right)\cos\delta+(B_2-B_5-B_6-2B_8)\sin\delta+B_1$
$-B_2\left(\frac{\delta\cos\delta+\sin\delta}{2}\right)-B_3\left(\frac{-\delta\sin\delta+\cos\delta}{2}\right)$
$+B_4\left(\frac{-\delta^2\cos\delta+\delta\sin\delta-3\cos\delta}{4}\right)+B_5\left(\frac{\delta^2\sin\delta+\delta\cos\delta+3\sin\delta}{4}\right)$
$+B_6\delta+B_7\left(\frac{\delta^3\sin\delta}{6}+\frac{\delta^2\cos\delta}{4}-\frac{\delta\sin\delta}{4}+\frac{7}{4}\cos\delta\right)$
$+B_8\left(\frac{-\delta^3\cos\delta}{6}+\frac{\delta^2\sin\delta}{4}+\frac{\delta\cos\delta}{4}+\frac{7}{4}\sin\delta\right)=0.$

(c) $\left(+B_1-\frac{B_2}{2}-\frac{3}{4}B_4+\frac{7}{4}B_7\right)\sin\delta+(B_2-B_5-B_6-2B_8)\cos\delta$
$-B_2\left(\frac{-\delta\sin\delta+2\cos\delta}{2}\right)-B_3\left(\frac{-\delta\cos\delta-2\sin\delta}{2}\right)$
$+B_4\left(\frac{\delta^2\sin\delta-\delta\cos\delta+4\sin\delta}{4}\right)+B_5\left(\frac{\delta^2\cos\delta+\delta\sin\delta+4\cos\delta}{4}\right)$
$+B_6+B_7\left(\frac{\delta^3\cos\delta}{6}+\frac{\delta^2\sin\delta}{4}+\frac{\delta\cos\delta}{4}-2\sin\delta\right)$
$+B_8\left(\frac{\delta^3\sin\delta}{6}-\frac{\delta^2\cos\delta}{4}+\frac{\delta\sin\delta}{4}+2\cos\delta\right)$
$+\frac{a}{\sin\phi\cos\phi}\Big[-\left(A_1-\frac{A_2}{2}-\frac{3}{4}A_4-\frac{K_1\sigma a}{\sin\phi}M_{t_A}\right)\sin\delta$
$-\left(A_2-A_5-A_6+K_1a(1+\sigma)M_{n_A}\right)\cos\delta+A_2\left(\frac{2\cos\delta-\delta\sin\delta}{2}\right)$
$+A_3\left(\frac{-2\sin\delta-\delta\cos\delta}{2}\right)-A_4\left(\frac{\delta^2\sin\delta-\delta\cos\delta+4\sin\delta}{4}\right)$
$-A_5\left(\frac{\delta^2\cos\delta+\delta\sin\delta+4\cos\delta}{4}\right)-A_6$
$-\sin^2\phi\left\{\left(-A_1+\frac{A_2}{2}+\frac{3}{4}A_4+\frac{K_1\sigma a}{\sin\phi}M_{t_A}\right)\sin\delta\right.$
$+(-A_2+A_5+A_6-K_1a(1+\sigma)M_{n_A})(\cos\delta-1)+A_1\delta$
$-A_2\frac{\delta\sin\delta}{2}-A_3\frac{\delta\cos\delta}{2}+A_4\left(\frac{-\delta^2\sin\delta-3\delta\cos\delta}{4}\right)$
$\left.+A_5\left(\frac{-\delta^2\cos\delta+3\delta\sin\delta}{4}\right)+A_6\frac{\delta^2}{2}\right\}\Big]=0.$

(60)

(d) $C_4+C_5\sin\delta+C_6\cos\delta+a\delta\cot\phi(C_2\cos\delta-C_3\sin\delta)+wa^2\delta=0.$

(e) $C_5\cos\delta-C_6\sin\delta$
$+a\cot\phi\{C_2(\cos\delta-\delta\sin\delta)-C_3(\sin\delta+\delta\cos\delta)\}+wa^2+ma=0.$

(f) $-\cot\phi(C_5\sin\delta+C_6\cos\delta)+C_4\tan\phi+\delta a\cot^2\phi(-C_2\cos\delta+C_3\sin\delta)$
$-\frac{a}{\sin^2\phi}\left(C_2\sin\delta+C_3\cos\delta\right)-\frac{a}{\cos^2\phi}C_1-\delta wa^2\cot\phi=0.$

$$
\begin{aligned}
\text{(a)}\quad & -\varepsilon\sin^2\phi\left[\frac{\sin\delta}{\delta}(2-\tan^2\phi)-\cos\delta+\tan^2\phi-1\right]C_1\\
&+\left[\frac{\cos\delta-1}{\delta}(\sigma\cos^2\phi-\varepsilon_1\cos 2\phi)+\frac{\sin\delta}{4}(3\sigma\cos^2\phi-\varepsilon\sin^2\phi-3\varepsilon_1\cos 2\phi)\right.\\
&\left.+\delta\cos\delta(\varepsilon-\sigma)\frac{\sin^2 2\phi}{8}-\delta^2\sin\delta\frac{\cos^2\phi}{12}\varepsilon_3\right]C_2\\
&+\left[\frac{1}{4}(\cos^2\phi(5\sigma\sin^2\phi-1)+\varepsilon(\cos^2 2\phi+\sin^4\phi))\left(\cos\delta-\frac{\sin\delta}{\delta}\right)\right.\\
&\left.+\varepsilon_4\delta\sin\delta\frac{\sin 2\phi}{4}-\varepsilon_3\delta^2\cos\delta\frac{\cos^2\phi}{12}\right]C_3\\
&+\frac{\sin^2\phi}{a}\left[-\frac{\sin\delta}{\delta}(3\varepsilon_4+\varepsilon\tan\phi)+\varepsilon_4\cos\delta+2\varepsilon_4+\varepsilon\tan\phi\right]C_4\\
&+\frac{\sin^2\phi}{a}\left[-(2\varepsilon_4+\varepsilon\cot\phi)\frac{\cos\delta-1}{\delta}-\frac{\sin\delta}{4}(5\varepsilon_4+(1+3\varepsilon)\cot\phi)\right.\\
&\left.+\frac{\delta\cos\delta}{4}\varepsilon_3\cot\phi\right]C_5+\frac{\sin^2\phi}{4a}\left[\left(\frac{\sin\delta}{\delta}-\cos\delta\right)(3\varepsilon_3-2\varepsilon)\cot\phi\right.\\
&\left.-\delta\sin\delta\varepsilon_3\cot\phi\right]C_6+wa\frac{\sin 2\phi}{2}\left[\frac{\cos\delta-1}{\delta}(4\varepsilon_3-\varepsilon-2)+\varepsilon_3\sin\delta\right.\\
&\left.+\frac{\delta}{2}(2\sigma\sin^2\phi+\varepsilon\cos 2\phi)\right]+m(1+\sigma)\frac{\sin 2\phi}{2}\left(2\frac{\cos\delta-1}{\delta}+\sin\delta\right)=0.\\
\text{(b)}\quad & -\varepsilon\sin^2\phi[(1-\tan^2\phi)(\cos\delta-1)+\delta\sin\delta]C_1+\left[\varepsilon_1\sin^2\phi\left(\frac{\delta\cos\delta-\sin\delta}{2}\right)\right.\\
&\left.+\delta^2\sin\delta\frac{\cos^2\phi}{4}(\varepsilon_3-2\varepsilon-2)-\frac{\delta^2\cos\delta}{12}\varepsilon_3\cos^2\phi\right]C_2\\
&+\left[\frac{\delta\sin\delta}{4}(\cos^2\phi-\varepsilon-\varepsilon_1\sin^2\phi)+\frac{\delta^2\cos\delta}{4}(-\cos^2\phi-\varepsilon+\varepsilon_1\sin^2\phi)\right.\\
&\left.+\frac{\delta^2\sin\delta}{12}\varepsilon_3\cos^2\phi\right]C_3-[(\cos\delta-1)(2\varepsilon_4+\varepsilon\tan\phi)+\varepsilon_4\delta\sin\delta]\frac{\sin^2\phi}{a}C_4\\
&+[(\sin\delta-\delta\cos\delta)(3\varepsilon_3-2\varepsilon-4)-\varepsilon_3\delta^2\sin\delta]\frac{\sin 2\phi}{8a}C_5+[(\varepsilon_3-2\varepsilon)\delta\sin\delta\\
&-\varepsilon_3\delta^2\cos\delta]\frac{\sin 2\phi}{8a}C_6+wa\frac{\sin 2\phi}{2}[\sin\delta(-3\varepsilon_3+\varepsilon+2)+\varepsilon_3\delta\cos\delta\\
&+\delta(2\sigma\sin^2\phi+\varepsilon\cos 2\phi)]+m(1+\sigma)\frac{\sin 2\phi}{2}(\delta\cos\delta-\sin\delta)=0.\\
\text{(c)}\quad & +\varepsilon\sin^2\phi[\sin\delta(1-\tan^2\phi)+\delta(\tan^2\phi-\cos\delta)]C_1+\left[\frac{\delta\sin\delta}{4}\{\varepsilon_3(1+3\sin^2\phi)\right.\\
&\left.+2\cos^2\phi(\varepsilon_3-1)-2\varepsilon\}+\frac{\delta^2\cos\delta}{4}\cos^2\phi(\varepsilon_3-2\varepsilon-2)+\frac{\delta^2\sin\delta}{12}\varepsilon_3\cos^2\phi\right.\\
&\left.+\varepsilon_1(1-\cos\delta)\sin^2\phi-(1+\sigma)\cos\delta\right]C_2+\left[\frac{\sin\delta}{2}(\varepsilon_3-\varepsilon+\varepsilon_1\sin^2\phi)\right.\\
&\left.+\frac{\delta\cos\delta}{2}(\varepsilon_3-\varepsilon-\varepsilon_1\sin^2\phi)+\frac{\delta^2\sin\delta}{4}(\cos^2\phi+\varepsilon-\varepsilon_1\sin^2\phi)\right.\\
&\left.+\frac{\delta^2\cos\delta}{12}\varepsilon_3\cos^2\phi\right]C_3+\frac{\tan\phi}{a}\sin^2\phi[\sin\delta(2\varepsilon_1-\varepsilon)\\
&+\delta\cos\delta(\varepsilon-\varepsilon_3)-\delta\varepsilon_1]C_4+\frac{\tan\phi}{a}\left[\frac{\cos^2\phi}{4}\{\delta\sin\delta(3\varepsilon_3-2\varepsilon-4)-\varepsilon_3\delta^2\cos\delta)\right.\\
&\left.+(\cos\delta-1)\sin^2\phi(\varepsilon_1-\varepsilon)-(1+\sigma)\cos\delta\right]C_5+\frac{\tan\phi}{a}\left[\frac{\sin\delta}{4}\{\varepsilon_3+3(1+\sigma)\right.\\
&\left.+\cos^2\phi(\varepsilon_1-1)\}+\frac{\cos^2\phi}{4}\{\delta\cos\delta(\varepsilon_3-2\varepsilon)+\varepsilon_3\delta^2\sin\delta\}\right]C_6\\
&+wa\tan\phi[(1-\cos\delta)\{\varepsilon_3\sin^2\phi+(1+2\sin^2\phi)(\varepsilon_1-\varepsilon)\}\\
&-\delta\sin\delta.\varepsilon_3\cos^2\phi-\delta^2(\varepsilon_1-\varepsilon)\sin^2\phi-(1+\sigma)\cos\delta]\\
&-m(1+\sigma)\frac{\sin 2\phi}{2}(\delta\sin\delta+\cos\delta+\tan^2\phi)=0.\\
\text{(d)}\quad & C_4+C_5\sin\delta+C_6\cos\delta+a\delta\cot\phi(C_5\cos\delta-C_6\sin\delta)+wa^2\delta=0.\\
\text{(e)}\quad & C_5\cos\delta-C_6\sin\delta\\
&+a\cot\phi\{C_5(\cos\delta-\delta\sin\delta)-C_6(\sin\delta+\delta\cos\delta)\}+wa^2+ma=0.\\
\text{(f)}\quad & -\cot\phi(C_5\sin\delta+C_6\cos\delta)+\tan\phi C_6+\delta a\cot^2\phi(-C_5\cos\delta+C_6\sin\delta)\\
&-\frac{a}{\sin^2\phi}(C_5\sin\delta+C_6\cos\delta)-\frac{a}{\cos^2\phi}C_1-\delta wa^2\cot\phi=0.
\end{aligned}
\qquad (61)
$$

ensuring that equations (31) are satisfied. This is not possible in the present case, and it is therefore necessary to ensure that the reactions at the supports satisfy the equations of equilibrium ; that is, that the components of all external forces and moments, including the reactions, about three mutually perpendicular axes passing through the support B are equal to zero.

The external forces and moments are as shown in *Fig.* 16 and the equations of equilibrium are given by (62).

$$\left.\begin{aligned}
&\text{(a)}\ R_{x_A} + R_{x_B} = 0.\\
&\text{(b)}\ R_{y_A} + R_{y_B} = 0.\\
&\text{(c)}\ R_{z_A} + R_{z_B} + \int_0^\delta w.ds = 0.\\
&\text{(d)}\ \int_0^\delta m.ds.\sin\theta + M'_{x_A} + R_{z_A}(y_A - y_B)\\
&\qquad\qquad + \int_0^\delta w.ds.(y - y_B) - R_{y_A}(z_A - z_B) = 0.\\
&\text{(e)}\ -\int_0^\delta m.ds.\cos\theta + M'_{y_A} + R_{x_A}(z_A - z_B)\\
&\qquad\qquad - R_{z_A}(x_A - x_B) - \int_0^\delta w.ds.(x - x_B) = 0.\\
&\text{(f)}\ M'_{z_A} + R_{y_A}(x_A - x_B) - R_{x_A}(y_A - y_B) = 0.
\end{aligned}\right\} \quad (62)$$

in which R_{x_A}, R_{y_A}, R_{z_A}, M'_{x_A}, M'_{y_A}, M'_{z_A} and R_{x_B}, R_{y_B}, R_{z_B}, $M'_{x_B} = 0$, $M'_{y_B} = 0$, $M'_{z_B} = 0$ are the reactions at the two supports.

From equations (13) and (14), equations (63) are obtained.

$$\left.\begin{aligned}
&x_A - x_B = a\cos 0 - a\cos\delta = a(1 - \cos\delta)\ ;\\
&y_A - y_B = a\sin 0 - a\sin\delta = -a\sin\delta\ ;\\
&z_A - z_B = 0 - \delta a\cot\phi = -\delta a\cot\phi\ ;\\
&x - x_B = a(\cos\theta - \cos\delta)\ ;\quad y - y_B = a(\sin\theta - \sin\delta)\ ;\\
&\int_0^\delta w.ds = \int_0^\delta \frac{wa}{\sin\phi}d\theta = \frac{wa}{\sin\phi}.\delta\ ;\\
&\int_0^\delta w.ds(y - y_B) = \int_0^\delta \frac{wa^2}{\sin\phi}(\sin\theta - \sin\delta)d\theta = \frac{wa^2}{\sin\phi}[(1 - \cos\delta) - \delta\sin\delta];\\
&\int_0^\delta w.ds(x - x_B) = \int_0^\delta \frac{wa^2}{\sin\phi}(\cos\theta - \cos\delta)d\theta = \frac{wa^2}{\sin\phi}(\sin\delta - \delta\cos\delta)\ ;\\
&\int_0^\delta m.ds.\sin\theta = \int_0^\delta \frac{ma}{\sin\phi}\sin\theta.d\theta = \frac{ma}{\sin\phi}(1 - \cos\delta)\ ;\\
&\int_0^\delta m.ds.\cos\theta = \int_0^\delta \frac{ma}{\sin\phi}\cos\theta.d\theta = \frac{ma}{\sin\phi}\sin\delta.
\end{aligned}\right\} \quad (63)$$

Substituting these values in equations (62), the equations of equilibrium (64) are obtained.

The components of T_t, T_n, T_b, M_t, M_n, and M_b along the three axes O_x, O_y, O_z at any point are obtained from equations (65).

$$
\left.\begin{aligned}
&\text{(a)}\ R_{x_A} + R_{x_B} = 0.\\
&\text{(b)}\ R_{y_A} + R_{y_B} = 0.\\
&\text{(c)}\ R_{z_A} + R_{z_B} + \frac{wa}{\sin\phi}\delta = 0.\\
&\text{(d)}\ \frac{ma}{\sin\phi}(1-\cos\delta) + M'_{x_A} - a\sin\delta.R_{z_A} + \frac{wa^2}{\sin\phi}[(1-\cos\delta) - \delta\sin\delta]\\
&\qquad\qquad + a\cot\phi.\delta R_{y_A} = 0.\\
&\text{(e)}\ -\frac{ma}{\sin\phi}\sin\delta + M'_{y_A} - a\cot\phi.\delta R_{x_A} - a(1-\cos\delta)R_{z_A}\\
&\qquad\qquad - \frac{wa^2}{\sin\phi}(\sin\delta - \delta\cos\delta) = 0.\\
&\text{(f)}\ M'_{z_A} + a(1-\cos\delta)R_{y_A} + a\sin\delta.R_{x_A} = 0.
\end{aligned}\right\} \quad (64)
$$

$$
\left.\begin{aligned}
T_x &= \alpha T_t + lT_n + \lambda T_b & M_x &= \alpha M_t + lM_n + \lambda M_b\\
T_y &= \beta T_t + mT_n + \mu T_b & M_y &= \beta M_t + mM_n + \mu M_b\\
T_z &= \gamma T_t + nT_n + \nu T_b & M_z &= \gamma M_t + nM_n + \nu M_b
\end{aligned}\right\} \quad (65)
$$

in which α, β, γ ; l, m, n ; λ, μ, ν ; are the direction-cosines of the three principal axes T, N, and B ; for a helix these are given by equations (16). The components of the reactions at supports A and B in the directions of the axes O_x, O_y, O_z are as given in equations (66).

$$
\left.\begin{aligned}
&\text{At A } (\theta = 0): \quad R_{x_A} = T_{x_A},\ R_{y_A} = T_{y_A},\ R_{z_A} = T_{z_A},\\
&\qquad\qquad M'_{x_A} = M_{x_A},\ M'_{y_A} = M_{y_A},\ M'_{z_A} = M_{z_A}.\\
&\text{At B } (\theta = \delta):\\
&\quad R_{x_B} = -T_{x_B},\ R_{y_B} = -T_{y_B},\ R_{z_B} = -T_{z_B},\\
&\quad M'_{x_B} = -M_{x_B} = 0,\ M'_{y_B} = -M_{y_B} = 0,\ M'_{z_B} = -M_{z_B} = 0.
\end{aligned}\right\} \quad (66)
$$

This check is made in the following numerical example.

Example.

FIXED AT THE TOP AND PIN-JOINTED AT THE BOTTOM.—Consider the staircase shown in *Fig.* 6.

As before, $\sin\phi = 0{\cdot}735$, $\cos\phi = 0{\cdot}678$, $\tan\phi = 1{\cdot}085$, $\cot\phi = 0{\cdot}922$,

$w = 352$ lb. per ft., $m = 206$ ft.-lb. per ft. $\delta = \frac{4\pi}{3} = 4{\cdot}18879$ radians.

$\varepsilon = 0{\cdot}397$, $\sigma = 0{\cdot}957$, $\varepsilon_1 = 0{\cdot}654$, $\varepsilon_2 = 4{\cdot}31$, $\varepsilon_3 = 1{\cdot}70$, $\varepsilon_4 = 0{\cdot}279$.

Inserting these values in equations (61),

(a) $-0{\cdot}109C_1 + 0{\cdot}684C_2 + 0{\cdot}256C_3 + 0{\cdot}206C_4 + 0{\cdot}047C_5 + 0{\cdot}317C_6 = 804.$
(b) $+0{\cdot}722C_1 + 4{\cdot}083C_2 - 2{\cdot}775C_3 + 0{\cdot}462C_4 + 1{\cdot}119C_5 + 0{\cdot}496C_6 = -1279.$
(c) $+1{\cdot}541C_1 + 7{\cdot}612C_2 - 9{\cdot}879C_3 - 0{\cdot}601C_4 + 0{\cdot}876C_5 - 1{\cdot}781C_6 = -4554.$
(d) $0 - 5{\cdot}633C_2 + 9{\cdot}757C_3 + 1{\cdot}000C_4 - 0{\cdot}866C_5 - 0{\cdot}500C_6 = -12{,}555.$
(e) $0 + 8{\cdot}412C_2 + 7{\cdot}962C_3 + 0 \quad - 0{\cdot}500C_5 + 0{\cdot}866C_6 = -3598.$
(f) $-6{\cdot}353C_1 + 9{\cdot}866C_2 - 6{\cdot}294C_3 + 1{\cdot}085C_4 + 0{\cdot}798C_5 + 0{\cdot}461C_6 = +11{,}572.$

Hence $C_1 = -1565$; $C_2 = -191$; $C_3 = -1098$;
$C_4 = -3033$; $C_5 = -3481$; $C_6 = +5791$.

Inserting these in equations (24), the forces and moments are as in (67).

$$\left.\begin{aligned}
T_t &= -191\sin\theta - 1098\cos\theta + 947\theta - 1565 \\
T_n &= +1493\sin\theta - 260\cos\theta \\
T_b &= +176\sin\theta + 1012\cos\theta + 1027\theta - 1698 \\
M_t &= -3481\sin\theta + 5791\cos\theta - 514\theta\cos\theta + 2953\theta\sin\theta + 2997\theta - 3033 \\
M_n &= -3860\sin\theta - 5433\cos\theta + 4016\theta\cos\theta + 699\theta\sin\theta + 4893 \\
M_b &= +4239\sin\theta + 587\cos\theta + 476\theta\cos\theta - 2722\theta\sin\theta - 2763\theta + 6652
\end{aligned}\right\} \quad (67)$$

When $\theta = 0$, $T_{t_A} = -2663$ lb.; $T_{n_A} = -260$ lb.; $T_{b_A} = -686$ lb.;
$M_{t_A} = +2758$ ft.-lb.; $M_{n_A} = -540$ ft.-lb.; $M_{b_A} = -7239$ ft.-lb.

When $\theta = \frac{4\pi}{3}$, $T_{t_B} = +3116$ lb.; $T_{n_B} = -1163$ lb.; $T_{b_B} = +1946$ lb.;
$M_{t_B} = M_{n_B} = M_{b_B} = 0$.

Equations (67) are checked by means of the equations of equilibrium (64). From equations (16), when $\theta = 0$, $\alpha = 0$ $\beta = 0{\cdot}735$ $\gamma = 0{\cdot}678$
$l = -1$ $m = 0$ $n = 0$
$\lambda = 0$ $\mu = -0{\cdot}678$ $\nu = 0{\cdot}735$

and when $\theta = \frac{4\pi}{3}$ $\alpha = 0{\cdot}637$ $\beta = -0{\cdot}368$ $\gamma = 0{\cdot}678$
$l = 0{\cdot}5$ $m = 0{\cdot}866$ $n = 0$
$\lambda = -0{\cdot}587$ $\mu = 0{\cdot}339$ $\nu = 0{\cdot}735$.

From equations (65),

At A: $T_{x_A} = -T_{n_A} = +260$ lb.;
$T_{y_A} = 0{\cdot}735T_{t_A} - 0{\cdot}678T_{b_A} = -1493$ lb.;
$T_{z_A} = 0{\cdot}678T_{t_A} + 0{\cdot}735T_{b_A} = -2309$ lb.;
$M_{x_A} = -M_{n_A} = +540$ ft.-lb.;
$M_{y_A} = 0{\cdot}735M_{t_A} - 0{\cdot}678M_{b_A} = -2878$ ft.-lb.;
$M_{z_A} = 0{\cdot}678M_{t_A} + 0{\cdot}735M_{b_A} = +7192$ ft.-lb.

At B: $T_{x_B} = 0{\cdot}637T_{t_B} + 0{\cdot}50T_{n_B} - 0{\cdot}587T_{b_B} = +260$ lb.;
$T_{y_B} = -0{\cdot}368T_{t_B} + 0{\cdot}866T_{n_B} + 0{\cdot}339T_{b_B} = -1494$ lb.;
$T_{z_B} = 0{\cdot}678T_{t_B} + 0{\cdot}735T_{b_B} = +3543$ lb.;
$M_{x_B} = M_{y_B} = M_{z_B} = 0$.

From equations (66),

$R_{x_A} = +260$ lb., $R_{y_A} = -1493$ lb., $R_{z_A} = -2309$ lb.
$M'_{x_A} = +540$ ft.-lb., $M'_{y_A} = -2878$ ft.-lb., $M'_{z_A} = +7192$ ft.-lb.
$R_{x_B} = -260$ lb., $R_{y_B} = +1494$ lb., $R_{z_B} = -3543$ lb., $M'_{x_B} = M'_{y_B} = M'_{z_B} = 0$.

Inserting these in equations (64) gives $+260 - 260 = 0$; $-1493 + 1494 = 0$;

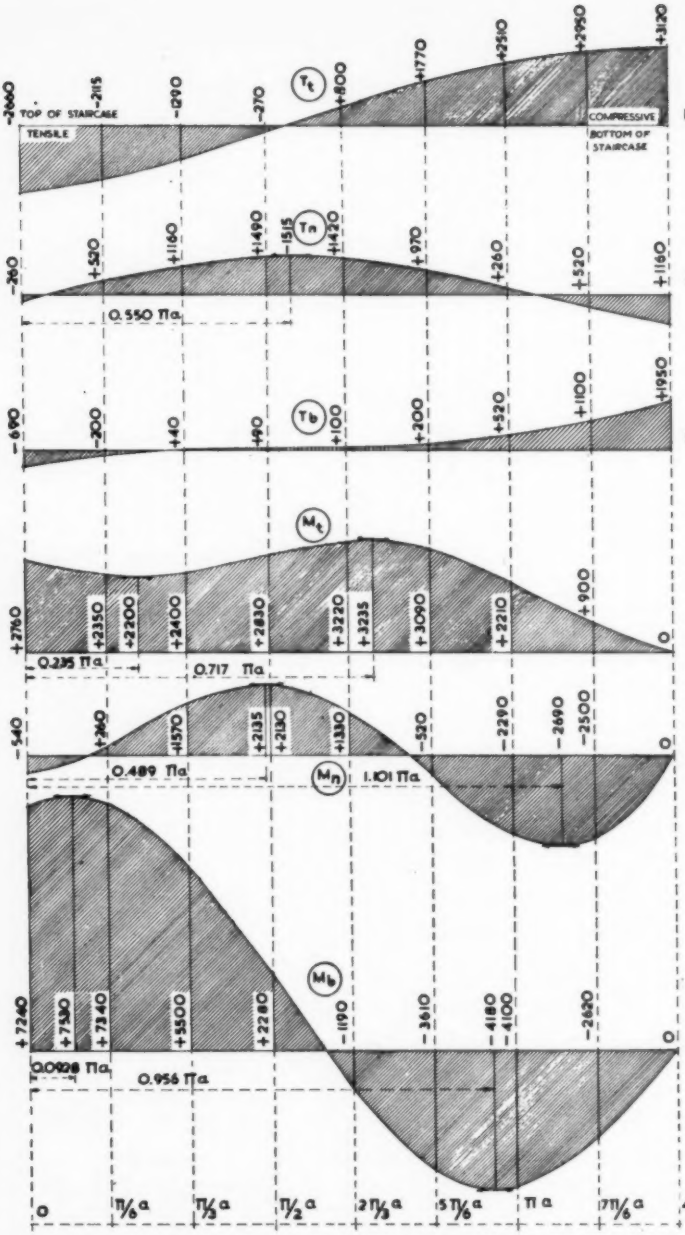


Fig. 17.—Diagram of Forces and Moments Fixed at the Top and Pinned at the Bottom.

$-2309 - 3543 + 5850 = 0$; $+1226 + 540 - 5835 + 20901 - 16831 = 0$; $+708 - 2878 - 2929 + 10106 - 2929 = 0$; and $+7192 - 6535 - 657 = 0$. The check is therefore satisfactory.

The maximum and minimum values are obtained by differentiation:

$\frac{dT_t}{d\theta} = -191\cos\theta + 1098 + 947 = 0$: as the roots are outside the limits there is no maximum or minimum.

$\frac{dT_n}{d\theta} = +1493\cos\theta + 260\sin\theta = 0$: when $\theta = 0{\cdot}550\pi$ $T_{n_{max.}} = +1515$ lb.

$\frac{dT_b}{\delta\theta} = 176\cos\theta - 1012\sin\theta + 1027 = 0$: as the roots are outside the limits there is no maximum or minimum.

$\frac{dM_t}{d\theta} = -3995\cos\theta - 2838\sin\theta + 514\,\theta\sin\theta + 2953\,\theta\cos\theta + 2997 = 0$

when $\theta = 0{\cdot}235\pi$ (M_t min. $= +2200$ ft.-lb.),
and when $\theta = 0{\cdot}717\pi$ (M_t max. $= +3235$ ft.-lb.).

$\frac{dM_n}{d\theta} = +156\cos\theta + 6132\sin\theta - 4016\,\theta\sin\theta + 699\,\theta\cos\theta = 0$

when $\theta = 0{\cdot}489\pi$ (M_n max. $= +2135$ ft.-lb.),
and when $\theta = 1{\cdot}101\pi$ (M_n min. $= -2690$ ft.-lb.).

$\frac{dM_b}{d\theta} = 4712\cos\theta - 3309\sin\theta - 473\,\theta\sin\theta - 2722\theta\cos\theta - 2763 = 0$

when $\theta = 0{\cdot}0928\pi$ (M_b max. $= +7530$ ft.-lb.),
and when $\theta = 0{\cdot}956\pi$ (M_b min. $= -4180$ ft.-lb.).

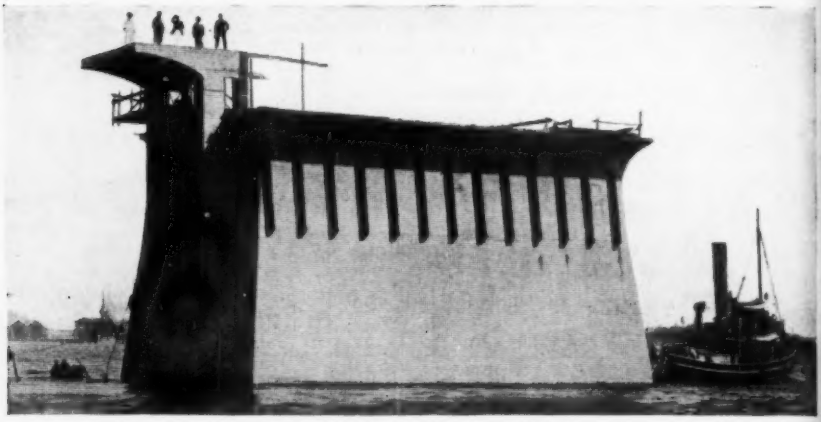
The diagrams of forces and moments are shown in *Fig.* 17. In accordance with the sign conventions of *Figs.* 1 to 5, normal tensile forces are assumed to be negative, and negative bending moments cause tension at the bottom and outer faces of the beam.

FIXED AT THE BOTTOM AND PIN-JOINTED AT THE TOP.—The procedure for finding the constants C_1 to C_6 is unchanged, but in equations (24) θ is replaced by $-\theta$ to obtain equations (24a) in which the origin is at the bottom.

$$\left.\begin{aligned}
T_t &= C_1 - C_2\sin\theta + C_3\cos\theta - \theta\,aw\cot\phi \\
T_n &= \frac{1}{\sin\phi}[C_2\cos\theta + C_3\sin\theta] \\
T_b &= \cot\phi(C_2\sin\theta - C_3\cos\theta) + C_1\tan\phi - aw.\theta \\
M_t &= C_4 - C_5\sin\theta + C_6\cos\theta - a\theta\cot\phi(C_2\cos\theta + C_3\sin\theta) - wa^2\theta \\
M_n &= \frac{1}{\sin\phi}[C_5\cos\theta + C_6\sin\theta + a\cot\phi\{C_2(\cos\theta - \theta\sin\theta) \\
&\qquad + C_3(\sin\theta + \theta\cos\theta)\} + wa^2 + ma] \\
M_b &= \cot\phi(C_5\sin\theta - C_6\cos\theta) + C_4\tan\phi + a\cot^2\phi.\theta(C_2\cos\theta + C_3\sin\theta) \\
&\quad + \frac{a}{\sin^2\phi}(C_2\sin\theta - C_3\cos\theta) - \frac{a}{\cos^2\phi}C_1 + \theta wa^2\cot\phi.
\end{aligned}\right\} \quad (24a)$$

FIFTY YEARS AGO.

From "CONCRETE AND CONSTRUCTIONAL ENGINEERING", September, 1909.



A REINFORCED concrete structure for torpedo trials has recently been installed in the Mediterranean Sea. The structure is a caisson 77 ft. by 55 ft. by 51 ft. high, with a platform overhanging a distance of 18 ft. at one side. The caisson was built in a dry dock in a period of five months, and was then floated and towed to Toulon Roads where it was anchored to a buoy while the internal work was completed. The draught was then 11 ft., and sufficient concrete was poured into the compartments to increase the draught to 26 ft. to ensure stability while it was towed a distance of 25 miles out to sea, where water was pumped into it to cause it to settle on to a prepared platform at a depth of 40 ft. The central compartments were then filled with sand and the outer compartments with concrete; sand was used in the central compartments so that it could be easily removed to lighten the structure should it be desired to tow it to another site.

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